ENG37

UNIVERSITY OF BOLTON

NATIONAL CENTRE FOR MOTORSPORT ENGINEERING

BEng (HONS) AUTOMOTIVE PERFORMANCE ENGINEERING (MOTORSPORT)

SEMESTER 1 EXAMINATION 2023/2024

ENGINEERING MATHEMATICS

MODULE NUMBER MSP4022

Date Wednesday 10th January 2024 **Time: 2:00pm – 4:00pm**

INSTRUCTIONS TO CANDIDATES

This paper has FIVE questions.
Answer ALL FIVE questions.

The maximum marks possible for each question and part question are shown in brackets.

Electronic calculators may be used if data and program storage memory is cleared prior to the examination.

Mobile phones or tablets may not be used as calculators.

There is a formula sheet at the back of the paper.

National Centre of Motorsports Engineering
BEng (Hons) Automotive Engineering (Motorsport)
Semester One Exam 2023/24
Engineering Mathematics
Module No. MSP4022

Question 1

a) Given the vectors $\boldsymbol{a} = 2\boldsymbol{i} - 3\boldsymbol{j} + 5\boldsymbol{k}$ and $\boldsymbol{b} = 7\boldsymbol{i} + 2\boldsymbol{j} - 6\boldsymbol{k}$ find the angle between the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$.

(5 marks)

b) Find the moment of force $\boldsymbol{F}$ about the point O, as depicted in Figure 1.

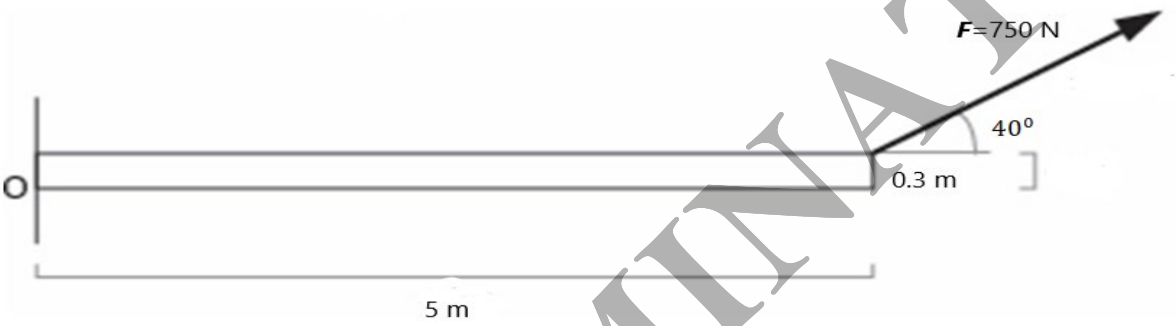

Figure 1

(15 marks)

Total for Question 1 (20 marks)

Question 2

a) If $\boldsymbol{A} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 4 & 0 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 4 & -12 & 7 \\ -3 & 9 & -5 \\ 1 & -2 & 1 \end{pmatrix}$, find $\boldsymbol{AB}$ and, without performing further calculations, write down $\boldsymbol{BA}$.

(10 marks)

b) Given the following pair of simultaneous linear equations:

$$2u_1 + u_2 = 4$$
$$u_1 - 3u_2 = -5$$

write these in the matrix form $\boldsymbol{Ku} = \boldsymbol{f}$, find $\boldsymbol{K}^{-1}$ and hence find u_1 and u_2.

(10 marks)

Total for Question 2 (20 marks)

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Question 3

Given the two complex numbers $z_1 = -2 - 3j$ and $z_2 = 2 - 4j$:

a) Display z_1 and $\overline{z_1}$ on an Argand diagram.

(2 marks)

b) Find:

i) $2z_1 - 3z_2$.

ii) $z_1 z_2$.

iii) $z_1 \overline{z_1}$.

(6 marks)

c) Working to 2 decimal places, convert z_1 and z_2 to polar form and hence find:

i) $\frac{z_1}{z_2}$

ii) ${z_1}^2$

(8 marks)

d) Find the complex roots of the quadratic equation:

$$x^2 + 2x + 5 = 0$$

(4 marks)

Total for Question 3 (20 marks)

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Question 4

a) Differentiate $y = x^3$ from first principles:

(6 marks)

b) Calculate the derivative of the following functions:

i) $y = e^{3x} \sin 4x$

(3 marks)

ii) $y = \cos(x^2)$

(3 marks)

c) Find and classify the stationary points of the following function:

$$y = x^3 + 3x^2 - 24x + 11$$

(8 marks)

Total for Question 4 (20 marks)

Question 5

Evaluate the following integrals:

i) $\int (3x^2 - 2x + 1)\, dx$

(4 marks)

ii) $\int_0^{\pi} (3\cos 3x - 2\sin 2x)\, dx$

(6 marks)

iii) $\int_1^3 te^{-3t}\, dt$ giving your answer correct to 3 decimal places.

(10 marks)

Total for Question 5 (20 marks)

END OF QUESTIONS

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Formula Sheet

Quadratic Equations

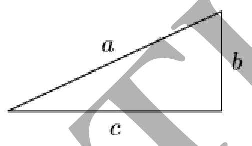
The equation:

$$ax^2 + bx + c = 0$$

has solutions:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Pythagoras' Theorem



$$a^2 = b^2 + c^2$$

Vectors

Vectors Given $\boldsymbol{a} = a_1\boldsymbol{i} + a_2\boldsymbol{j} + a_3\boldsymbol{k}$ and $\boldsymbol{b} = b_1\boldsymbol{i} + b_2\boldsymbol{j} + b_3\boldsymbol{k}$ then:

$$|\boldsymbol{a}| = \sqrt{{a_1}^2 + {a_2}^2 + {a_3}^2}$$

$$\boldsymbol{a} \cdot \boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3 = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta$$

$$\boldsymbol{a} \times \boldsymbol{b} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \boldsymbol{i}\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \boldsymbol{j}\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \boldsymbol{k}\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

where $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

Matrices

(2×2) Matrices

The determinant of a (2×2) matrix A is given by:

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \implies \det(A) = a_{11}a_{22} - a_{12}a_{21}.$$

The inverse of the (2×2) matrix A is given by:

$$A^{-1} = \frac{1}{\det(A)} \begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix}.$$

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Table of Derivatives and Integrals

In the table below, m, n are any real numbers.

$\int F(x)\,dx$	$F(x)$	$F'(x)$
$\int f(x)\,dx + \int g(x)\,dx$	$f(x)+g(x)$	$f'(x)+g'(x)$
$m\int f(x)\,dx$	$mf(x)$	$mf'(x)$
$mx + C$	m	0
$\frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$	x^n	nx^{n-1}
$\ln(x) + C$	$\frac{1}{x}$	$-\frac{1}{x^2}$
$\frac{1}{m}e^{mx} + C$	e^{mx}	me^{mx}
$x\ln(mx) - x + C$	$\ln(mx)$	$\frac{1}{x}$
$\frac{1}{m}\sin(mx) + C$	$\cos(mx)$	$-m\sin(mx)$
$-\frac{1}{m}\cos(mx) + C$	$\sin(mx)$	$m\cos(mx)$

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Calculus Rules – Differentiation

Product Rule: $$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

Quotient Rule: $$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{1}{v^2}\left[v\frac{du}{dx} - u\frac{dv}{dx}\right]$$

Chain Rule: $$\frac{d}{dx}\left[y\left(u(x)\right)\right] = \frac{dy}{du}\frac{du}{dx}$$

Rules of Integration

INTEGRATION BY PARTS: $$\int_{x=a}^{b} f(x)g'(x)\,dx = \Big[f(x)g(x)\Big]_{x=a}^{b} - \int_{x=a}^{b} f'(x)g(x)\,dx$$

Local Maxima and Minima of a Function

A curve defined by $y = f(x)$ in terms of some function f has *stationary points* where $f'(x) = 0$. These are then classified using the *Second Derivative Test*:

Let $x = a$ be a stationary point of $f(x)$ then:

$f''(a) > 0 \implies x = a$ is a local minimum

$f''(a) < 0 \implies x = a$ is a local maximum

$f''(a) = 0 \implies$ the test is inconclusive.

END OF FORMULA SHEET

END OF PAPER