ENG35

UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

MSC SYSTEMS ENGINEERING AND ENGINEERING MANAGEMENT

SEMESTER 1 EXAMINATION 2023/2024

SIGNAL PROCESSING

MODULE NO: EEM7011

Date: Thursday 11th January 2024 **Time: 2:00pm – 4:00pm**

INSTRUCTIONS TO CANDIDATES:

There are SIX questions.

Answer ANY FOUR questions.

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

Electronic calculators may be used provided that data and program storage memory is cleared prior to the examination.

CANDIDATES REQUIRE: **Tables of Laplace and Z Transforms (attached).**

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Question 1.

Assume there is an analogue signal, $x(t) = 4\sin(1000\pi t)$ shown in Fig.1. Convert the analogue signal into digital form by considering the following steps:

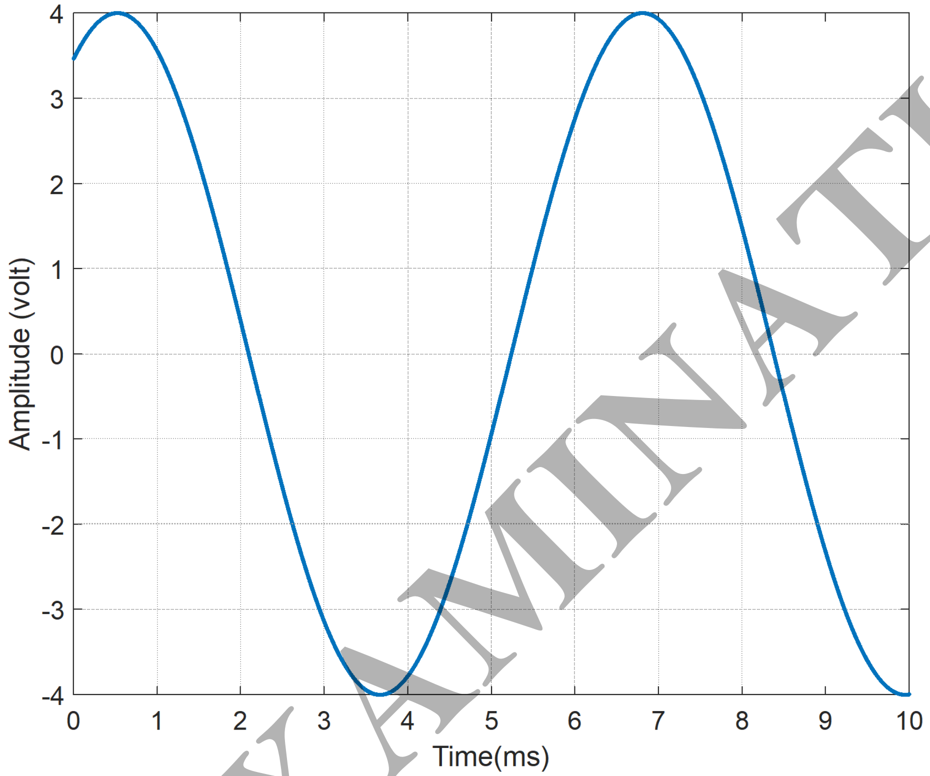

Fig.1: Showing an analogue signal.

a. Convert $x(t)$ into a discrete-time signal, $x(n)$ by assuming $f_s = 2f_m$, where f_s and f_m are the sampling and information frequency, respectively.
(8 marks)

b. Quantise $x(n)$ into the closest pre-defined level values by considering n = 3 bits
(7 marks)

c. Encode the quantised signal.
(8 marks)

d. Construct the total digital signal based on the individual codes in c.
(2 marks)

Total 25 marks

Question 2.

a) What does mean by signal processing and why do we use it?. (7 marks)

b) Give three advantages and three disadvantages of using digital signal processors (DSP) compared with their analogue counterparts. (6 marks)

c) Select the correct choice.

1) A filter that described by y(n) = Kx(n), if 0 < K<1, then the filter will works as (2 marks)

i. an amplifier
ii. an attenuator
iii. an isolator
iv. an inverter

2) The sum of an even signal and an odd signal is (2 marks)

i. even
ii. odd
iii. neither odd and nor even
iv. random

3) A continuous time signal is said to be odd if (2 marks)

i. it is identical to its continuous amplitude scale.
ii. it is identical to original point.
iii. it is identical to its continuous time-reversed versionn.
iv. it is identical around both time and attitude axes.

4) The signal transformation applied to $y[n] = x[n * 2]$ is (2 marks)

i. time shifting.
ii. time scaling (compression).
iii. amplitude scaling.
iv. time scaling (expansion).

Question 2 continues over the page
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Question 2 continued

5) The signal transformation applied to y[n]= x[-n] is

(2 marks)

i. time reversal.
ii. time scaling.
iii. amplitude scaling.
iv. time shifting.

6) The signal transformation applied to $y(\mathrm{t})$=$x(\mathrm{t}+\mathrm{a})$ + B is:

(2 marks)

i. combined time scaling time shifting.
ii. combined time scaling and amplitude shifting.
iii. combined time shifting and amplitude shifting.
iv. combined time scaling and time reversal.

Total 25 marks

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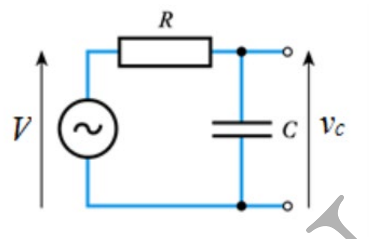
Question 3.

a) Use Laplace Transform Properties to find the transfer function, H(s), of the RC filter below

(9 marks)



b) Find the Laplace inverse of the system described by following formula:

$$H(s) = \frac{s - 6}{s^2 - s - 6}$$

(6 marks)

c) Determine the roots of the poles and zeros of the system transfer function below and then plot them on the s-plane. Is the system stable or unstable, give vertification for your answer.

$$\frac{5(s+2)(s+3)}{(s+4)(s^2 + 6s + 13)}$$

(10 marks)

Total 25 marks
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Question 4.

a) Find out the Z-transform & Region of Convergence (ROC) for the following sequence.

$$x_1[n] = \left(\frac{1}{2}\right)^n u[n] + \left(\frac{1}{4}\right)^n u[n]$$

(12marks)

b) Determine the inverse z-transform of the following function:

$$X(z) = \frac{3 + 2z^{-1} + z^{-2}}{1 - 3z^{-1} + 2z^{-2}}$$

(13 marks)

Total 25 marks

Question 5.

A digital filter is defined by the formula below

$$y(n) = 1.625\, y(n-1) - 0.934\, y(n-2) + 0.5\, x(n) - 0.1x(n-2)$$

a) State the order of the filter, type of the filter and identify the values of its coefficients

(9marks)

b) Draw a block diagram using Direct Form I structure

(8 marks)

c) Draw a block diagram using Direct Form II structure

(8 marks)

Total 25 marks

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Question 6.

a) A band pass filter is required to meet the following specification

- Complete signal rejection at dc and 500Hz
- A narrow pass band centred at 250Hz
- A 3dB bandwidth of 20Hz

By assuming the sampling frequency is 1000Hz,

1) Find the transfer function using the zero pole placement method (7 marks)
2) Find the difference equation (3 marks)
3) Draw the diagrammatical solution (5 marks)

b) Evaluate the DFT of the sequence {1, 0, 0, 1} using the decimation in time FFT. (10 marks)

Total 25 marks

END OF QUESTIONS

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Laplace Transform Tables

	$f(t)$	$F(s)$	
1.	$\delta(t)$	1	Impulse function
2.	$u(t)$	$\frac{1}{s}$	Step function
3.	$t\,u(t)$	$\frac{1}{s^2}$	Ramp function
4.	$t^n\,u(t)$	$\frac{n!}{s^{n+1}}$	
5.	$e^{-at}\,u(t)$	$\frac{1}{s+a}$	
6.	$\sin \omega t\,u(t)$	$\frac{\omega}{s^2+\omega^2}$	
7.	$\cos \omega t\,u(t)$	$\frac{s}{s^2+\omega^2}$	

	$f(t)$	$F(s)$
	$df(t)/dt$	$sF(s)-f(0)$
	$df^2(t)/dt^2$	$s^2F(s)-sf(0)-f'(0)$

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Table of Laplace Transforms

	$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$		$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$
1.	1	$\frac{1}{s}$	2.	$\mathbf{e}^{at}$	$\frac{1}{s-a}$
3.	$t^n,\quad n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	4.	$t^p,\ p>-1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6.	$t^{n-\frac{1}{2}},\quad n=1,2,3,\ldots$	$\frac{1\cdot 3\cdot 5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7.	$\sin(at)$	$\frac{a}{s^2+a^2}$	8.	$\cos(at)$	$\frac{s}{s^2+a^2}$
9.	$t\sin(at)$	$\frac{2as}{(s^2+a^2)^2}$	10.	$t\cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
11.	$\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$	12.	$\sin(at)+at\cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
13.	$\cos(at)-at\sin(at)$	$\frac{s(s^2-a^2)}{(s^2+a^2)^2}$	14.	$\cos(at)+at\sin(at)$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$
15.	$\sin(at+b)$	$\frac{s\sin(b)+a\cos(b)}{s^2+a^2}$	16.	$\cos(at+b)$	$\frac{s\cos(b)-a\sin(b)}{s^2+a^2}$
17.	$\sinh(at)$	$\frac{a}{s^2-a^2}$	18.	$\cosh(at)$	$\frac{s}{s^2-a^2}$
19.	$\mathbf{e}^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	20.	$\mathbf{e}^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21.	$\mathbf{e}^{at}\sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	22.	$\mathbf{e}^{at}\cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23.	$t^n\mathbf{e}^{at},\quad n=1,2,3,\ldots$	$\frac{n!}{(s-a)^{n+1}}$	24.	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$

	$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$		$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$
25.	$u_c(t)=u(t-c)$ Heaviside Function	$\frac{\mathbf{e}^{-cs}}{s}$	26.	$\delta(t-c)$ Dirac Delta Function	$\mathbf{e}^{-cs}$
27.	$u_c(t)f(t-c)$	$\mathbf{e}^{-cs}F(s)$	28.	$u_c(t)g(t)$	$\mathbf{e}^{-cs}\mathfrak{L}\{g(t+c)\}$
29.	$\mathbf{e}^{ct}f(t)$	$F(s-c)$	30.	$t^n f(t),\quad n=1,2,3,\ldots$	$(-1)^n F^{(n)}(s)$
31.	$\frac{1}{t}f(t)$	$\int_s^{\infty} F(u)\,du$	32.	$\int_0^t f(v)\,dv$	$\frac{F(s)}{s}$
33.	$\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	34.	$f(t+T)=f(t)$	$\frac{\int_0^T \mathbf{e}^{-st}f(t)\,dt}{1-\mathbf{e}^{-sT}}$
35.	$f'(t)$	$sF(s)-f(0)$	36.	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
37.	$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)\cdots-sf^{(n-2)}(0)-f^{(n-1)}(0)$			

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Z Transform Table

Entry number	Discrete-time sequence $x(n),\quad n \geq 0$	z-transform $X(z)$	Region of convergence of $X(z)$
1	$k\delta(n)$	k	Everywhere
2	k	$\frac{kz}{z-1}$	$\lvert z\rvert > 1$
3	kn	$\frac{kz}{(z-1)^2}$	$\lvert z\rvert > 1$
4	kn^2	$\frac{kz(z+1)}{(z-1)^3}$	$\lvert z\rvert > 1$
5	$ke^{-\alpha n}$	$\frac{kz}{z-e^{-\alpha}}$	$\lvert z\rvert > e^{-\alpha}$
6	$kne^{-\alpha n}$	$\frac{kze^{-\alpha}}{(z-e^{-\alpha})^2}$	$\lvert z\rvert > e^{-\alpha}$
7	$1-e^{-\alpha n}$	$\frac{z(1-e^{-\alpha})}{z^2-z(1+e^{-\alpha})+e^{-\alpha}}$	$\lvert z\rvert > e^{-\alpha}$
8	$\cos(\alpha n)$	$\frac{z(z-\cos\alpha)}{z^2-2z\cos\alpha+1}$	$\lvert z\rvert > 1$
9	$\sin(\alpha n)$	$\frac{z\sin\alpha}{z^2-2z\cos\alpha+1}$	$\lvert z\rvert > 1$
10	$e^{-\alpha n}\sin(\alpha n)$	$\frac{ze^{-\alpha}\sin\alpha}{z^2-2e^{-\alpha}z\cos\alpha+e^{-2\alpha}}$	$\lvert z\rvert > e^{-\alpha}$
11	$e^{-\alpha n}\cos(\alpha n)$	$\frac{ze^{-\alpha}(ze^{\alpha}-\cos\alpha)}{z^2-2ze^{-\alpha}\cos\alpha+e^{-2\alpha}}$	$\lvert z\rvert > e^{-\alpha}$
12	$\cosh(\alpha n)$	$\frac{z^2-z\cosh\alpha}{z^2-2z\cosh\alpha+1}$	$\lvert z\rvert > \cosh\alpha$
13	$\sinh(\alpha n)$	$\frac{z\sinh\alpha}{z^2-2z\cosh\alpha+1}$	$\lvert z\rvert > \sinh\alpha$
14	$k\alpha^n$	$\frac{kz}{z-\alpha}$	$\lvert z\rvert > \alpha$
15	$kn\alpha^n$	$\frac{k\alpha z}{(z-\alpha)^2}$	$\lvert z\rvert > \alpha$
16	$2\lvert c\rvert\,\lvert p\rvert^n\cos(n\angle p+\angle c)$	$\frac{cz}{z-p}+\frac{c^*z}{z-p^*}$	

k and α are constants; c is a complex number.

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