[ENG30]

UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

B.ENG (HONS) MOTORSPORT ENGINEERING

SEMESTER ONE EXAMINATION 2023/2024

MECHANICS OF MATERIALS AND MACHINES

MODULE NO: AME5012

Date: Thursday 11th January 2024 **Time: 14:00 – 16:00**

INSTRUCTIONS TO CANDIDATES:

There are FIVE questions.

Answer FOUR questions.

Marks for parts of questions are shown in brackets.

Electronic calculators may be used provided that data and program storage memory is cleared prior to the examination.

CANDIDATES REQUIRE:

Formula Sheets (attached after questions).

Q1. Beam deflection

A cantilever beam of length L shown in **Figure Q1** is used in aerospace to support a wing of a military aircraft. It has to carry a uniformly distributed load, w, which includes the weight of the beam and a factor of safety of 3.5 has to be applied.

Given: $E = 70\ GPa$, $L = 3m$, $w = 50\ kN/m$.

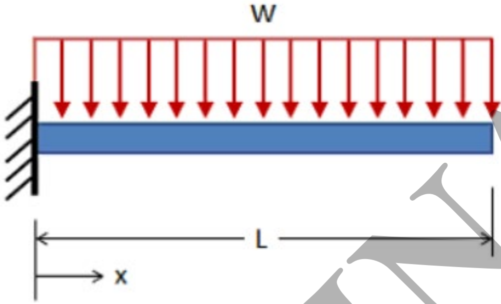

Figure Q1

a) Calculate the flexural rigidity (EI) of the beam if the maximum allowable deflection is not to exceed $2\ cm$.

[6 marks]

b) Determine the diameter of the cross-section beam, d , if it has a solid circular cross section.

[8 marks]

c) Calculate the maximum bending moment and the maximum bending stress.

[7 marks]

d) Is the beam safe if the yield stress σ_{yield} of the material used for the manufacturing process is $350\ MPa$?

[4 marks]

Total 25 Marks

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Q2. Thick cylinder

A thick cylinder has an outside diameter of $100\ mm$ and an inside diameter of $60\ mm$.

It is pressurised $120\ MPa$ on the inside surface.

If $E = 210\ GPa$ and $v = 0.27$ - Calculate:

a) The pressure difference between the inside and outside walls of the cylinder.

[7 marks]

b) The circumferential stress on the outside and on the inside.

[5 marks]

c) The longitudinal stress.

[3 marks]

d) The circumferential strain in the inside and the outside layer.

[5 marks]

e) The change in inner and outside diameter.

[5 marks]

Total 25 Marks

Q3. Struts

A straight steel alloy bar of thickness $4\,mm$ and width $10\ mm$ is axially loaded until buckling occurs. The steel alloy bar is free at both ends.

Given: The yield stress is $350\ MN/m^2$.

a) Calculate the length of the bar when $E = 70\ GN/m^2$ and the Euler buckling load is $80\ N$.

[6 marks]

b) Calculate the radius of gyration of the bar.

[5 marks]

c) Calculate the slenderness ratio of the bar. Assess the validity of the Euler formula based on this calculated value.

[7 marks]

d) Using Rankine-Gordon Strut theory, determine the maximum load that the straight aluminium alloy bar can support without buckling.

[7 marks]

Total 25 Marks

Q4: Principal Stresses

Test procedures on an aluminium engine block have indicated the following stress state in the x and y directions: 90 MPa and 50 MPa in tension. Due to thickening of the section at the position of interest there are possibilities of a shear stress present, one related to xy with a value of 30 MPa.

a) Draw the square element showing the stresses acting on it.

[4 marks]

b) Calculate the principal stresses using Formula Sheet.

[5 marks]

c) Calculate the principal stresses using Eigenvalue Matrix and compare them with the results from Q4b.

[6 marks]

d) Calculate the principal stresses using Mohr cycle and compare them with the results from Q4b.

[6 marks]

e) If the proof stress of the material in tension is $350\ MPa$ and the material follows the von Mises yield criterion, determine the factor of safety associated with this point in the block.

[4 marks]

Total 25 Marks

Q5: Application of Beam theory and FEA

A part of a car frame can be simplified into a clammed-clammed beam as shown in **Figure Q5**. The length, L, breath, b, height, h and thickness, t of the beam are 1.5 m, 10 mm 20 mm and 2 mm, respectively. The beam is made up of Aluminum Alloy 6063-T6 and the properties of which are given in **Table Q5**. The beam is under a uniformly distributed load (UDL), w of 560 N/m. The FEA technique was run (see the results at the appendix).

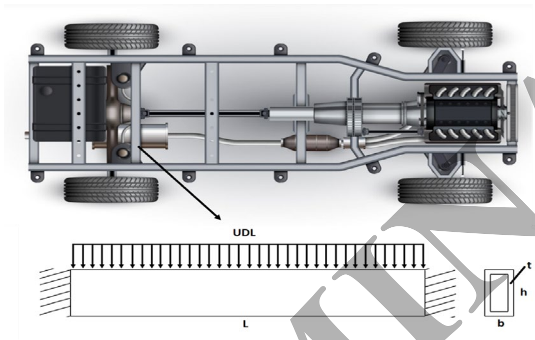

Figure Q5: Car Frame containing a clammed-clammed beam under UDL.

Table Q5: Properties of Aluminium Alloy 6063-T6

Young's Modulus, E	69 GPa
Poisson's ratio, ν	0.33
Yield Strength, σ_{ys}	220
Ultimate Strength, σ_{UTS}	250

a) Find out the maximum shear force, $F_{s(max)}$ using analytical technique, and compare the given results from the FEA technique.

[4 marks]

b) Find out the maximum bending moment, M_{max} using analytical technique, and compare the given results from the FEA technique.

[4 marks]

b) Find out the maximum bending stress, σ_{max} using analytical technique, and compare the given results from the FEA technique.

[6 marks]

c) Find the maximum deflection, D_{max} using analytical technique, and compare the given results from the FEA technique.

[6 marks]

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Question 5 continued…

d) Determine the safety factors SF_1 and SF_2 for Yield Strength and Ultimate Strength of the material, respectively. Comment on the results.

[5 marks]

Total 25 Marks

Equations:

Second Moment of area,

$$I = \frac{bh^3}{12} - \frac{(b-2t)(h-2t)^3}{12}$$

Perpendicular distance to the centroid,

$$y = \frac{h}{2}$$

Maximum Shear Force,

$$F_{s(max)} = \frac{wL}{2}$$

Maximum Bending Moment,

$$M_{max} = \frac{wL^2}{12}$$

Maximum Bending Stress,

$$\sigma_{max} = \frac{M_{max}y}{I}$$

Maximum Deflection,

$$D_{max} = \frac{wL^4}{384EI}$$

Safety Factors,

$$SF_1 = \frac{\sigma_{ys}}{\sigma_{max}} \qquad SF_2 = \frac{\sigma_{UTS}}{\sigma_{max}}$$

Total 25 Marks

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Question 5 continued…

Appendix: FEA results

FEA Results	
Maximum Shear Force(magnitude), F_{max}(N)	425
Maximum Bending Moment(magnitude), M_{max} (N.m)	110
Maximum Bending Stress, σ_{max}(Pa)	$225*10^6$
Maximum Deflection, D_{max}(mm)	22

END OF QUESTIONS

FORMULA SHEETS FOLLOW OVER THE PAGE….

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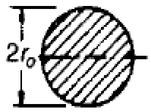
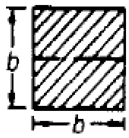
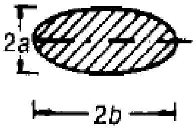
FORMULA SHEET

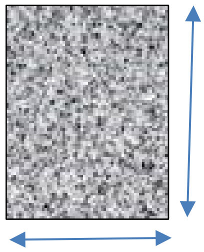
Deflection:

$$M_{xx} = EI \frac{d^2y}{dx^2}$$

Section Shape	$A(m^2)$	$I_{xx}(m^4)$
	πr^2	$\frac{\pi}{4} r^4$
	b^2	$\frac{b^4}{12}$
	πab	$\frac{\pi}{4} a^3 b$

For solid rectangular Cross-section

$$I_{xx} = \frac{bd^3}{12}$$



Plane Stress:

a) **Stresses in function of the angle Θ:**

$$\sigma_x(\theta) = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos(2\theta) + \tau_{xy}\sin(2\theta)$$

$$\sigma_y(\theta) = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos(2\theta) - \tau_{xy}\sin(2\theta)$$

$$\tau_{xy}(\theta) = -\frac{\sigma_x - \sigma_y}{2} sin(2\theta) + \tau_{xy} cos(2\theta)$$

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b) <u>Principal stresses:</u>

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$\tau_{\max} = \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

<u>Lame's equation</u>

The equations are known as "Lame's Equations" for radial and hoop stress at any specified point on the cylinder wall. Note: R_1 = inner cylinder radius, R_2 = outer cylinder radius

$$\sigma_C = a + \frac{b}{r^2}$$

$$\sigma_R = a - \frac{b}{r^2}$$

The corresponding strains format is:

$$\varepsilon_c = 1/E\ \{\sigma_c - \nu(\sigma_r + \sigma_L)\}$$

$$\varepsilon_r = 1/E\ \{\sigma_r - \nu(\sigma_c + \sigma_L)\}$$

$$\varepsilon_L = 1/E\ \{\sigma_L - \nu(\sigma_c + \sigma_r)\}$$

$$\sigma_L = \frac{P_1 R_1^2 - P_2 R_2^2}{(R_2^2 - R_1^2)}$$

$$\tau_{max} = \frac{\sigma_c - \sigma_r}{2} = \frac{b}{r^2}$$

Stress

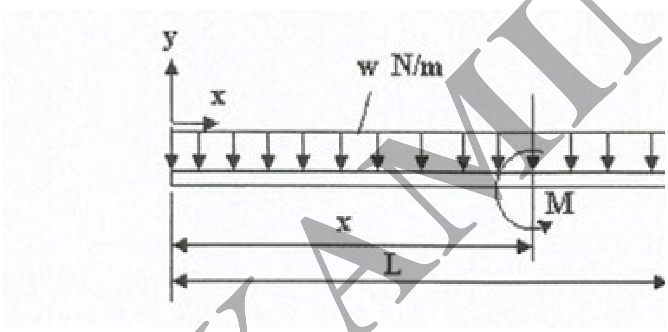
σ = Force/Area = F/A

Hook's law

σ = E·ε

ε = ∆L/L

Cantilever beam with UDL:



M: maximum bending moment ($M_{max}=\omega L^2/2$)

Maximum bending stress:

$$\sigma_{bending} = \frac{My}{I}$$

M: maximum bending moment
Y: distance from neutral axis
I: second moment of area

Slope at the ends:

$$\frac{dy}{dx} = \frac{\omega L^3}{6EI}$$

Maximum deflection at the middle:

$$y = -\frac{\omega L^4}{8EI}$$

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Yield Criterion

Von Mises

$$\sigma_{von\ Mises} = \frac{1}{\sqrt{2}}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2}$$

Quadratic equation: $ax^2+bx+c=0$

Solution:

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Allowable stress: $\boldsymbol{\sigma_{allowable}}$

$$\boldsymbol{\sigma_{allowable} = \frac{\sigma_{yield}}{Factor\ Of\ Safety}}$$

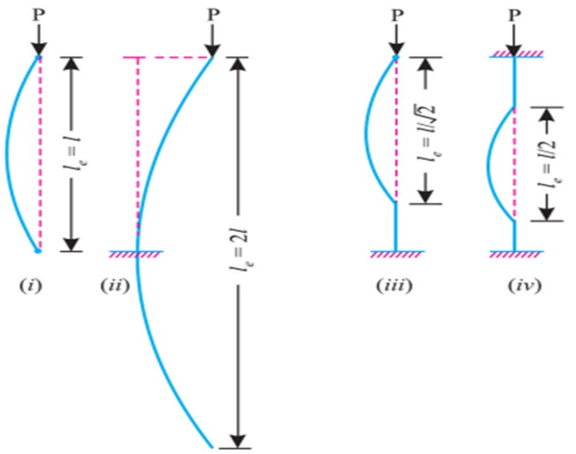
Struts:

$$I = k^2 A$$

$$k = \sqrt{\frac{I}{A}}$$

<u>Euler validity</u>

$$Slenderness\ ratio = SR = \frac{L_e}{k} \geq \pi\sqrt{\frac{E}{\sigma_{yield}}}$$



(i) Both ends pin jointed or hinged or rounded or free.
(ii) One end fixed and other end free.
(iii) One end fixed and the other pin jointed.
(iv) Both ends fixed.

Case	*End conditions*	*Equivalent length, l_e*	*Buckling load, Euler*
1	*Both ends hinged or pin jointed* or *rounded or free*	l	$\frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 EI}{l^2}$
2.	*One end fixed, other end free*	$2l$	$\frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 EI}{4l^2}$
3.	*One end fixed, other end pin jointed*	$\frac{l}{\sqrt{2}}$	$\frac{\pi^2 EI}{l_e^2} = \frac{2\pi^2 EI}{l^2}$
4.	*Both ends fixed or encastered*	$\frac{l}{2}$	$\frac{\pi^2 EI}{l_e^2} = \frac{4\pi^2 EI}{l^2}$

Studying Rankine's formula,

$$P_{Rankine} = \frac{\sigma_c \cdot A}{1 + a \cdot \left(\frac{l_e}{k}\right)^2}$$

We find,

$$P_{Rankine} = \frac{\text{Crushing load}}{1 + a\left(\frac{l_e}{k}\right)^2}$$

The factor $1 + a\left(\frac{l_e}{k}\right)^2$ has thus been introduced to *take into account the buckling effect.*

$$\boldsymbol{a = \frac{\sigma_c}{\pi^2 \cdot E}}$$

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