

UNIVERSITY OF BOLTON
NATIONAL CENTRE FOR MOTORSPORT
ENGINEERING
BEng (HONS) AUTOMOTIVE PERFORMANCE
ENGINEERING (MOTORSPORT)
SEMESTER 2 EXAMINATION 2022/2023
ADVANCED VEHICLE SYSTEMS
MODULE NUMBER MSP6011

Date Tuesday 9th May 2023

Time: 2:00 – 4:00pm

INSTRUCTIONS TO CANDIDATES

This paper has FIVE questions
The marks for each question are shown in brackets
Attempt ALL questions

Electronic calculators may be used provided that data and program storage memory is cleared prior to the examination

Mobile telephones or cellular telephones may-not be used as calculators

Formula sheet attached

Question 1

Figure Q1 shows a quarter car model of a vehicle suspension system.

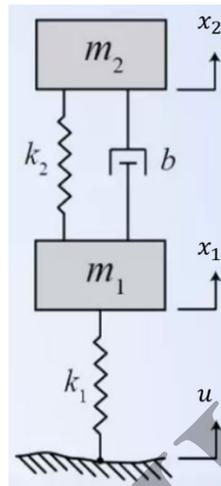


Figure Q1

- (a) Develop a system of differential equations describing the motion of the system.

(12 marks)

- (b) Determine the transfer function $G(s) = \frac{X_1(s)}{U(s)}$

(8 marks)

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Question 2

(a) A system is described by the equations:

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

Where:

$$A = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad C = [1 \quad 1]$$

Calculate the output transfer function $\frac{Y(s)}{U(s)}$

(10 marks)

(b) Develop a state space model (A, B, C and D matrices) for the following system:

$$m\ddot{x} + b\dot{x} + kx = f$$

With input $u = f$ and output $y = f - b\dot{x} - kx$

(10 marks)

Question 3

A DC motor can be represented as a first order system with a time constant of 0.5s.

When the input voltage is subject to a step increase from 0v to 8v, the speed of the motor increases from 0rpm to 6000rpm.

a) Develop a transfer function that represents the time response of the motor.

(5 marks)

b) Calculate the time taken for the motor to reach 90% of its final value of 6,000rpm.

(7.5 marks)

c) Calculate the speed of the motor 0.25s after the input voltage is subject to step increase from 0v to 1v

(7.5 marks)

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Question 4

Figure Q4 shows a block diagram representation of a linear automotive control system where $R(s)$ is the input, $D(s)$ is a disturbance and $Y(s)$ is the output.

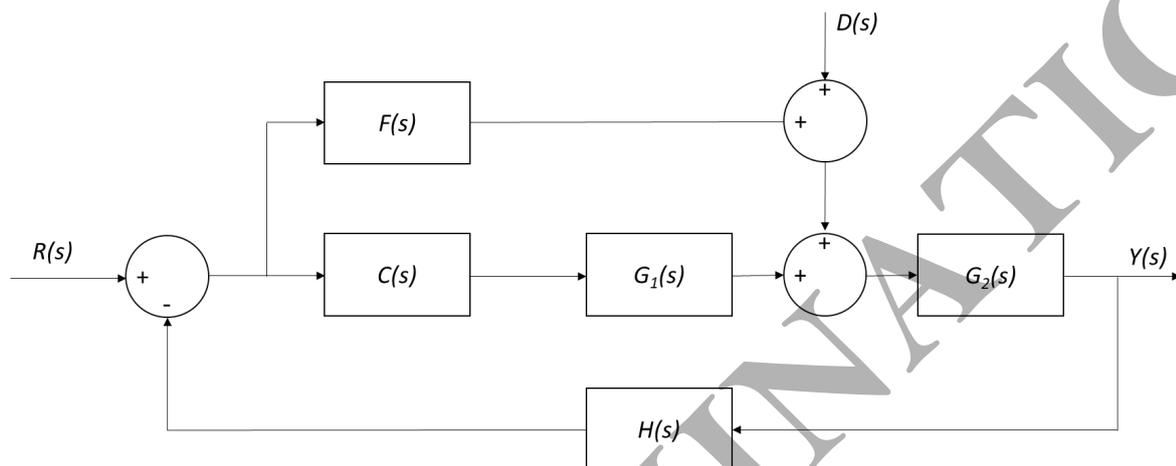


Figure Q4

a) Using block diagram reduction techniques, find the following transfer functions for the system:

I. $\frac{Y(s)}{R(s)}$ (10 marks)

II. $\frac{Y(s)}{D(s)}$ (10 marks)

Question 5

A system is represented by the following transfer function:

$$G(s) = \frac{100}{s + 30}$$

a) Sketch the Bode plots (magnitude and phase) of the system. (15 marks)

b) With reference to the Bode plots, describe the frequency response of the system. (5 marks)

END OF QUESTIONS
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FORMULA SHEET

Matrices:

For a 2x2 matrix:

$$\text{If } \mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

then

$$|\mathbf{A}| = ad - bc$$

$$\text{adj}\mathbf{A} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

And

$$\mathbf{A}^{-1} = \frac{\text{adj}\mathbf{A}}{|\mathbf{A}|}$$

1st Order Systems:

$$\tau \dot{y}(t) + y(t) = ku(t)$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{k}{\tau s + 1}$$

2nd Order Systems:

$$\ddot{y}(t) + 2\zeta\omega_n\dot{y}(t) + \omega_n^2y(t) = \omega_n^2u(t)$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

State Space:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t)$$

$$y(t) = \mathbf{B}\mathbf{x}(t) + \mathbf{D}u(t)$$

$$G(s) = \frac{Y(s)}{U(s)} = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D}$$

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