

**UNIVERSITY OF BOLTON**

**SCHOOL OF ENGINEERING**

**B.ENG (HONS) ELECTRICAL AND ELECTRONIC  
ENGINEERING**

**SEMESTER TWO EXAMINATION - 2022/2023**

**INSTRUMENTATION AND CONTROL**

**MODULE NO: EEE5011**

Date: Thursday 11<sup>th</sup> May 2023

Time: 14:00 – 16:30

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**INSTRUCTIONS TO CANDIDATES:**

There are SIX questions.

Answer ANY FOUR questions.

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

Electronic calculators may be used provided that data and program storage memory is cleared prior to the examination.

**CANDIDATES REQUIRE:**

Formula Sheet (attached).

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**Question 1**

A second order system has the following transfer function:

$$\frac{400}{s^2 + 25s + 400}$$

- (i) Find the natural frequency  $\omega_n$ , the damping ratio  $\zeta$  and the damped frequency  $\omega_d$ . **[6 marks]**
- (ii) Calculate the peak time and the rise time for the system **[6 marks]**
- (iii) Calculate the percentage overshoot for the system. **[3 marks]**

(b) A second order system has the following transfer function:

$$\frac{600}{s^2 + 50s + 600}$$

State, with reasons, whether the system is *underdamped* or *overdamped*.

**[2 marks]**

Find the response of the system to a unit step input.

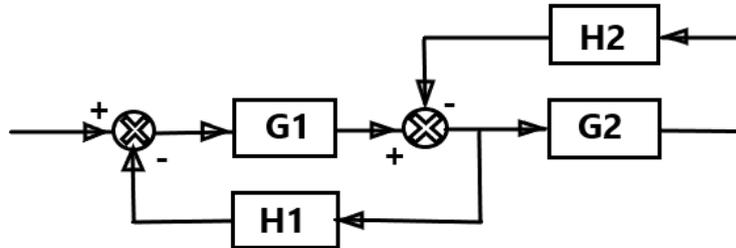
**[8 marks]**

**Total 25 marks**

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**Question 2**

Consider the system block diagram shown in Figure 2a.

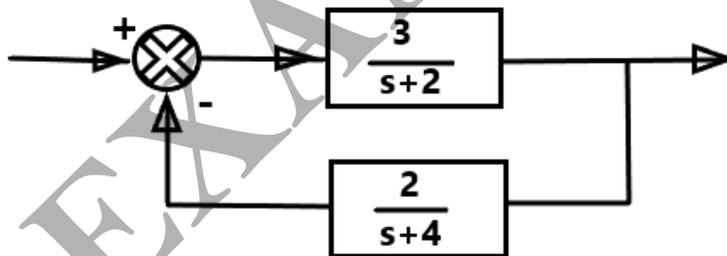


**Figure 2a**

By applying the rules for block reduction, find the transfer function to represent this system as a single block.

**[15 marks]**

(b) Consider the system block diagram shown in Figure 2b.



**Figure 2b**

Calculate and simplify the closed loop transfer function for the system.

**[10 marks]**

**Total 25 marks**

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### Question 3

Consider the control system shown in figure 3

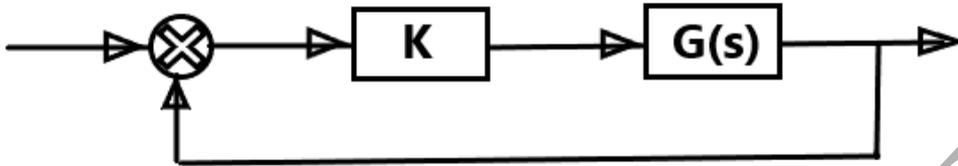


Figure 3

The controller  $K$  is a proportional controller, the plant has the following transfer function:

$$G(s) = \frac{2}{s^2 + 8s + 10}$$

and the system uses unity negative feedback. We wish to design the controller so that the overshoot does not exceed 20% and the steady state error for a unit step input does not exceed 0.15.

- (i) Write down the *open loop* transfer function of the system. **[2 marks]**
- (ii) Find an expression for the steady state error for unit step input in terms of  $K$ . **[4 marks]**
- (iii) Find the range of values for  $K$  for which the steady state error does not exceed 0.15. **[4 marks]**
- (iv) Find the *closed* loop transfer function of the system. **[3 marks]**
- (v) Find expressions for the natural frequency and the damping ratio in terms of  $K$ . **[5 marks]**
- (vi) Find the value of the damping ratio that gives an overshoot of 20%. **[3 marks]**
- (vii) Find the range of values for  $K$  for which the overshoot does not exceed 20%. **[4 marks]**

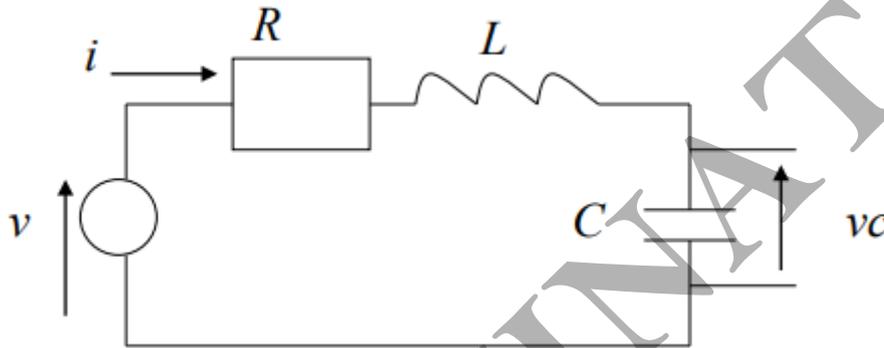
**Total 25 marks**

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**Question 4**

(a) For the system shown in Figure Q4a below, obtain:

- (i) the transfer function  $\frac{V_C(s)}{V(s)}$  [8 marks]
- (ii) the damping ratio [3 marks]
- (iii) the undamped natural angular frequency [3 marks]

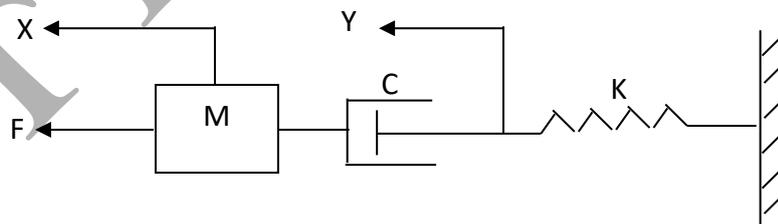


**Figure Q4a**

Where  $R=40 \Omega$  ,  $C =10 \mu F$  ,  $L=20 \text{ mH}$

(b) For the system shown in Figure Q4b below Y is the output and F is the input, obtain:

- i. The output-input differential equation. [5 marks]
- ii. the damping factor [3 marks]
- iii. the undamped natural angular frequency [3 marks]



**Figure Q4b**

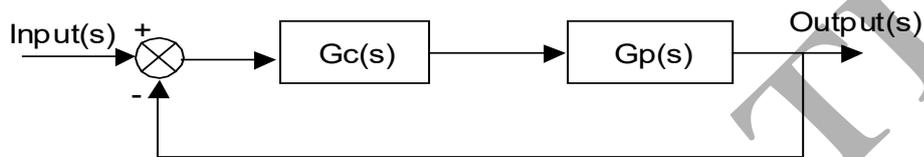
Where  $M = 12 \text{ kg}$ ,  $C = 1 \text{ Ns/m}$ ,  $K = 5 \text{ N/m}$

**Total 25 marks**

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**Question 5**

Consider the control system shown in Figure Q5 with its closed loop form.

**Figure Q5**

Where  $G_p(s) = \frac{1}{s(s^2 + 7s + 12)}$

- (a) If  $G_c(s)$  is a proportional controller ( $K_i = K_d = 0$ ), find the range of the gain  $K_p$  making the system to be an underdamped system. **[7 Marks]**

Find the  $K_i$  for a unit parabolic input ( $\frac{1}{s^3}$ ) if  $G_c(s)$  is a PI controller and the steady state error is less than 0.01. **[9 marks]**

- (b) The designer needs to achieve less than 20% overshoot and settling time less than 5 seconds. Design a PID controller by determining  $K_p$  and  $K_d$  (using the  $K_i$  obtained from (b) above) to satisfy these requirements. **[9 marks]**

**Total 25 marks**

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**Question 6**

(a) Enumerate three features that a medical measurement equipment should demonstrate regardless of the nature of data measured. [ 5 marks]

(b) List the static characteristics of a medical instrument and define clearly two of them. [ 6 marks]

(c) What are the main types of biomedical measurands? [ 6 marks]

Explain the function of a capacitive proximity sensor using the parameters of the capacitance formula  $C = \frac{\epsilon A}{x}$ . Illustrate your answer with the help of diagrams.

[8 marks]

**Total 25 marks**

**END OF QUESTIONS**

**Formula sheet follows over the page**

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### Formula sheet

These equations are given to save short-term memorisation of details of derived equations and are given without any explanation or definition of symbols; the student is expected to know the meanings and usage.

#### Parameters of underdamped second order systems

Relation between  $\omega_n$ ,  $\omega_d$  and  $\zeta$ :

$$\omega_d = \sqrt{1 - \zeta^2} \omega_n \quad \omega_n = \frac{1}{\sqrt{1 - \zeta^2}} \omega_d \quad \zeta = \sqrt{1 - \left(\frac{\omega_d}{\omega_n}\right)^2}$$

Relation between damping ratio and percentage overshoot:

$$\text{overshoot} = 100 \exp\left(-\frac{\zeta\pi}{\sqrt{1 - \zeta^2}}\right) \quad \zeta = \frac{\sqrt{(\ln A)^2}}{\pi^2 + (\ln A)^2}$$

Rise time, peak time, and 5% and 2% settling times:

$$t_{\text{rise}} = \frac{\pi - \phi}{\omega_d} \quad \text{where } \phi = \cos^{-1}(\zeta) \quad t_{\text{peak}} = \frac{\pi}{\omega_d}$$

$$t_{\text{settle},5\%} \approx \frac{3}{\zeta\omega_n} \quad t_{\text{settle},2\%} \approx \frac{4}{\zeta\omega_n}$$

#### Table of Laplace Transforms

$f(t)$	$F(s) = \int_0^{\infty} f(t)e^{-st} dt$
1	$\frac{1}{s}$
$t$	$\frac{1}{s^2}$
$e^{-at}$	$\frac{1}{s + a}$
$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
$e^{-at} f(t)$	$F(s + a)$
$f'(t)$	$sF(s) - f(0)$

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Block Diagram Algebra

Rule	Original Diagram	Equivalent Diagram
1. Moving a summing point beyond a block		
2. Moving a summing point in front a block		
3. Moving a takeoff point to front of a block		
4. Moving a takeoff point to beyond a block		

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Blocks with Feedback Loop

$$G(s) = \frac{Go(s)}{1 + Go(s)H(s)} \text{ (for a negative feedback)}$$

$$G(s) = \frac{Go(s)}{1 - Go(s)H(s)} \text{ (for a positive feedback)}$$

Steady State Error

$$e_{ss} = \lim_{s \rightarrow 0} [s(1 - G_o(s))\theta_i(s)] \text{ (for an open-loop system)}$$

$$e_{ss} = \lim_{s \rightarrow 0} [s \frac{1}{1 + G_o(s)} \theta_i(s)] \text{ (for the closed-loop system with a unity feedback)}$$

$$e_{ss} = \lim_{s \rightarrow 0} [s \frac{1}{1 + \frac{G_1(s)}{1 + G_1(s)[H(s) - 1]}} \theta_i(s)] \text{ (if the feedback } H(s) \neq 1)$$

$$e_{ss} = \lim_{s \rightarrow 0} [-s \cdot \frac{G_2(s)}{1 + G_2(G_1(s) + 1)} \cdot \theta_d] \text{ (if the system subjects to a disturbance input)}$$

First Order System

$$G(s) = \frac{\theta_o}{\theta_i} = \frac{G_{ss}(s)}{\tau s + 1}$$

$$\tau \left( \frac{d\theta_o}{dt} \right) + \theta_o = G_{ss} \theta_i$$

$$\theta_o = G_{ss} (1 - e^{-t/\tau}) \text{ (for a unit step input)}$$

$$\theta_o(t) = G_{ss} [t - \tau(1 - e^{-(t/\tau)})] \text{ (for a unit ramp input)}$$

$$\theta_o(t) = G_{ss} \left( \frac{1}{\tau} \right) e^{-(t/\tau)} \text{ (for an impulse input)}$$

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Second order system

$$\frac{d^2\theta_o}{dt^2} + 2\zeta\omega_n \frac{d\theta_o}{dt} + \omega_n^2\theta_o = b_o\omega_n^2\theta_i$$

$$G(s) = \frac{\theta_o(s)}{\theta_i(s)} = \frac{b_o\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

## Standard PID controllers

- *Proportional only:*  $G_P(s) = K_P$
- *Proportional plus Integral:*  
 $G_{PI}(s) = K_p + K_i/s = K_p(1 + 1/\tau_i s)$

Where  $\tau_i = K_p/K_i$

- *Proportional plus derivative:*  
 $G_{PD}(s) = K_p + K_D s = K_p(1 + \tau_d s)$

Where  $\tau_d = K_d / K_p$

- *Proportional, integral and derivative:*  
 $G_{PID}(s) = K_p + K_i/s + K_d s = K_p(1 + 1/\tau_i s + \tau_d s)$

END OF PAPER