ENG36

# UNIVERSITY OF BOLTON

# SCHOOL OF ENGINEERING

# BENG (HONS) MECHANICAL ENGINEERING

# SEMESTER 2 EXAMINATION 2022/2023

# FINITE ELEMENT & DIFFERENCE METHODS

# MODULE N$^{O}$: AME6016

**Date: Thursday 11th May 2023** **Time: 10:00 – 12:00pm**

**INSTRUCTIONS TO CANDIDATES:**

**There are FOUR questions**

**Attempt ANY THREE questions.**

**All questions carry equal marks.**
**75 marks equates to 100%**

**Marks for parts of questions are shown in brackets.**

**Formula Sheet is attached in the APPENDIX at the end of the paper**

**Q1**

a) A stepped shaft with a thrust collar is represented by 3 x 1D finite elements manufactured from steel with a design stress of 300 MPa and an elastic modulus of 205GPa. Take each element as 50 mm long, elements 1 and 3 have a 400mm$^2$ cross-section area, element 2 has a cross-sectional area of 800mm$^2$ and they are connected in series as shown below in Fig. Q1a.

Using the FEM and 1D elements, determine:

(i) the maximum load F that can be applied at the step change,

**(ii)** the maximum displacement and the strain in element 2**.**

(18 Marks)

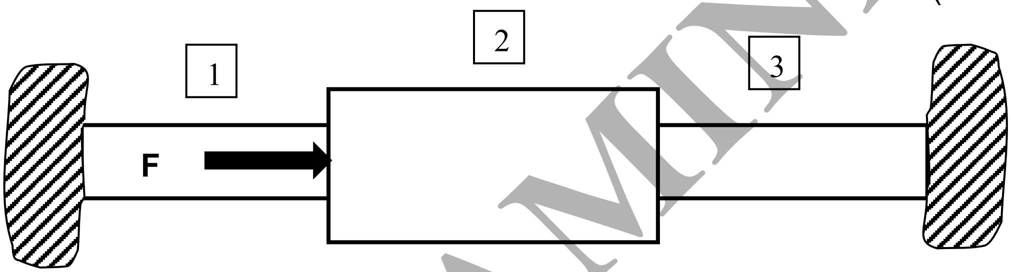


**Fig Q1a Schematic set of FEA model**

b) Figs Q1b, c and d shows a course FEA mesh, a stress plot and a strain energy density plot respectively for a component under cyclic loading. Describe briefly and utilizing a sketch how the mesh would change if mesh adaption was used based upon the H method. If the P method was used instead explain briefly why the mesh could remain the same and how the convergence solution would be obtained based upon the strain energy being the target function.

(7 marks)

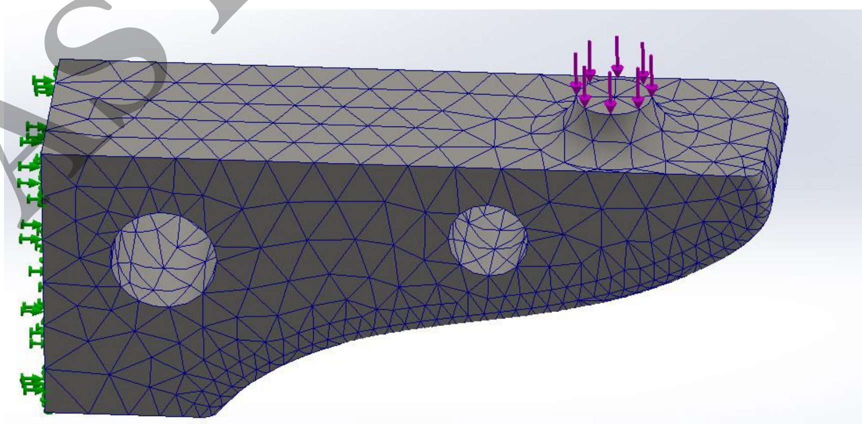

**Fig Q1b course FEA mesh**

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**Q1 continued**

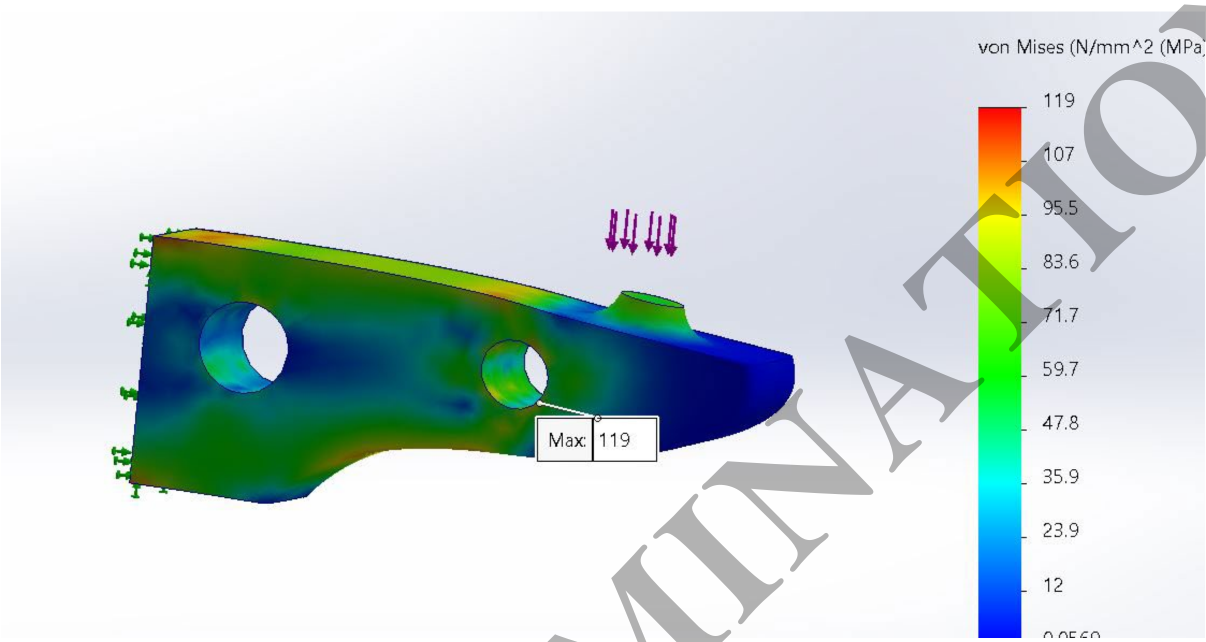


**Fig Q1c stress plot**

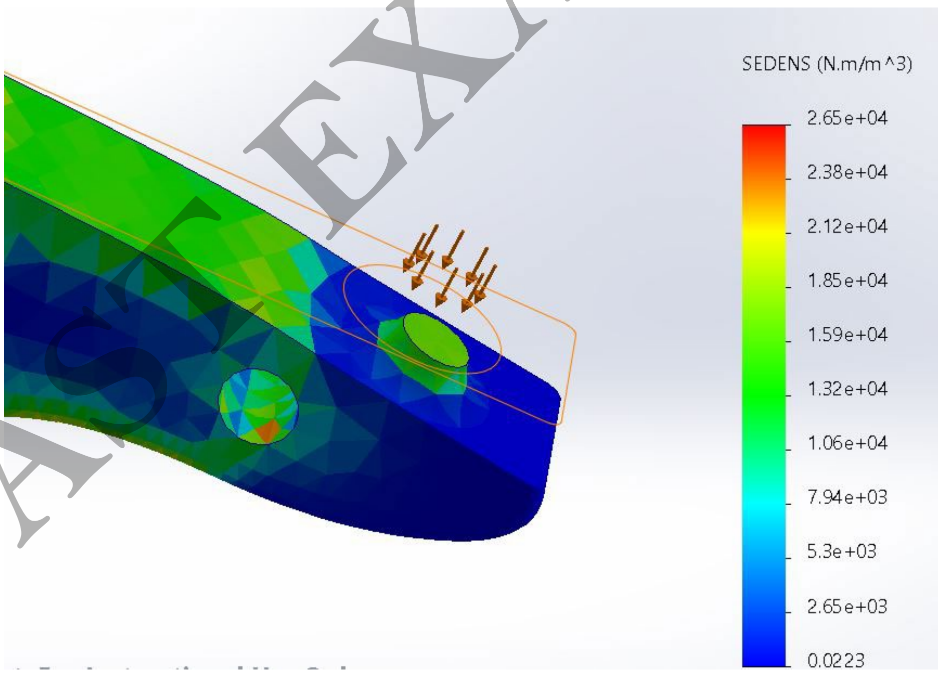


**Fig Q1d strain energy density plot**

**Total 25 Marks**

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**Q2**

A simple suspension support system can be modelled as shown schematically in Fig Q2. The main beam is L in length with a mass M which is seven times that of the support spring situated at the left-hand side. Take the stiffening spring (K) stiffness to be equivalent to $3EI/L^3$.

Using the usual notation determine an expression for the first two natural frequencies in terms of E, I, A, L and ρ including any constants derived. State any assumptions you have made.

(14 marks)

Also, determine an expression for the mode shape associated with the first mode in terms of L and the shafts local coordinate x and sketch the shape of this mode.

(6 marks)

Briefly explain the difference between the lumped mass and consistent mass approaches to solving vibration problems using FEA. Also explain why this analysis uses the implicit methodology (5 marks)

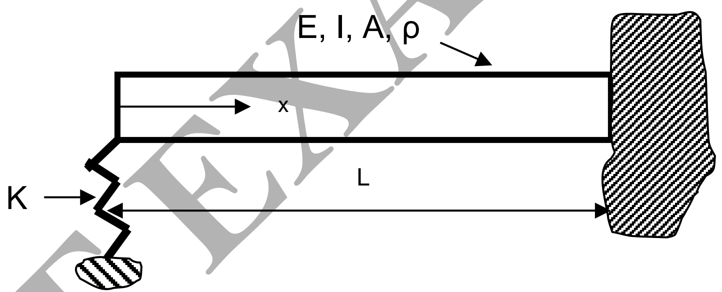


**Fig Q2 Schematic model**

**Total 25 Marks**

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**Q3: Stress analysis in thick pressure vessel using FDM**

A pressure vessel is being tested in the laboratory to check its ability to withstand pressure for a submarine engineering application. The thick pressure vessel has an inner radius a = 13cm and an outer radius b = 21cm. The pressure vessel is made out of an ASTM A350 steel with a yield strength of 350 MPa.

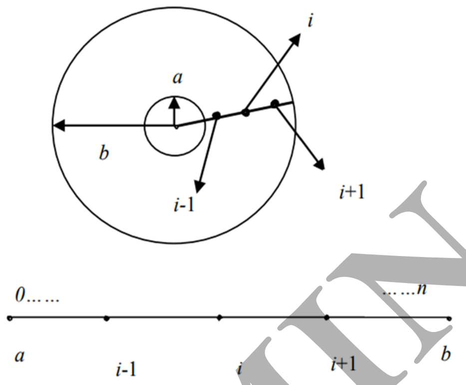


*Figure 3: Nodes along the radial*

Two strain gages that are bonded tangentially at the inner and the outer radius measure normal tangential strain at the maximum needed pressure as $\in_{t\,/\,r=a}$ = 0.00077462 and $\in_{t\,/\,r=b}$ = 0.00038462

Since the radial displacement and tangential strain are related simply by $\in_t = u / r$, the maximum normal stress in the pressure vessel is at the inner radius r = a is given by:

$$\sigma_{max} = \frac{E}{1-\nu^2}\left(\frac{u}{r} + \nu\frac{du}{dr}\right)$$

Where E is the Young's modulus of the steel (E= 230 GPa) and v is the Poisson's ratio (v = 0.3)

Also, the radial thickness of the pressure vessel is divided into 5 equidistant nodes from r=a to r=b to apply the FDM calculations. Therefore, radial displacements corresponding to the nodes are given by u1, u2, u3, u4 and u5.

**Q3 continues over the page**

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**Q3 continued**

a) Calculate u1 and u5 analytically at the boundary of the pressure vessel.
(3 marks)

b) Find the maximum normal stress and the factor of safety, given that the other values of the radial displacements are u2 = 0.00009335, u3 = 0.0000907 and u4 = 0.00008935.
(8 marks)

c) Find the exact value of the maximum normal stress knowing that the analytical equation for a radial displacement is given by:

$$u = C_1 r + \frac{C_2}{r}$$

$C_1$ and $C_2$ are 2 constants depending to the boundary conditions. (10 marks)

d) Calculate the relative and the true error.
(4 marks)

**Total 25 Marks**

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**Q4: 1D steady-state heat diffusion (conduction) in a bar using FDM**

The following equation called a transport equation, as the temperature (representing the thermal energy of the fluid) is transported by the motion of the fluid (U), can be solved to compute the temperature T of the fluid:

$$\frac{\partial(\rho c_p T)}{\partial t} + \underbrace{\nabla \cdot (\rho c_P \boldsymbol{U} T)}_{\text{Convection}} = \underbrace{\nabla \cdot (k \nabla T)}_{\text{Diffusion}} + S$$

Where $C_p$ is the specific heat capacity of the fluid, k is the thermal conductivity of the fluid and S is a heat source (per unit fluid volume). *Note that thermal energy can also be transported by radiation, but this will not be considered here.*

**Consider 1D steady-state diffusion of heat in a jet engine shaft bar**, as shown in Figure 4. The bar is also **insulated from the surrounding ambiance temperature** and it has a length of 5m, a cross-sectional area of 0.1 $m^2$ and a thermal conductivity of 100 W/mK. The temperature at the left end of the bar ($T_1$) is 200◦C and the temperature at the right end ($T_5$) is 100◦C. There is a constant heat source of 1000 $W/m^3$ in the bar. The temperature field in the bar is governed by the 1D steady-state heat diffusion equation.

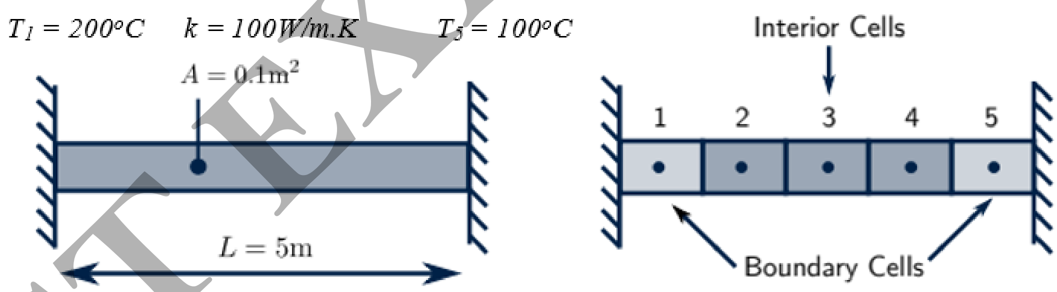


Figure 4: **Heat Diffusion in a Bar**

a) Give the analytical and the numerical 1D steady-state heat diffusion equation governing the temperature field in the bar applying the divergence theorem and the FDM technics.

(8 marks)

b) Evaluate the temperature in each cell and comment on the results.

(17 marks)

**Total 25 Marks**

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School of Engineering
BEng (Hons) Mechanical Engineering
Semester 2 – Examination 2022/2023
Finite Element & Difference Methods
Module N°. AME6016

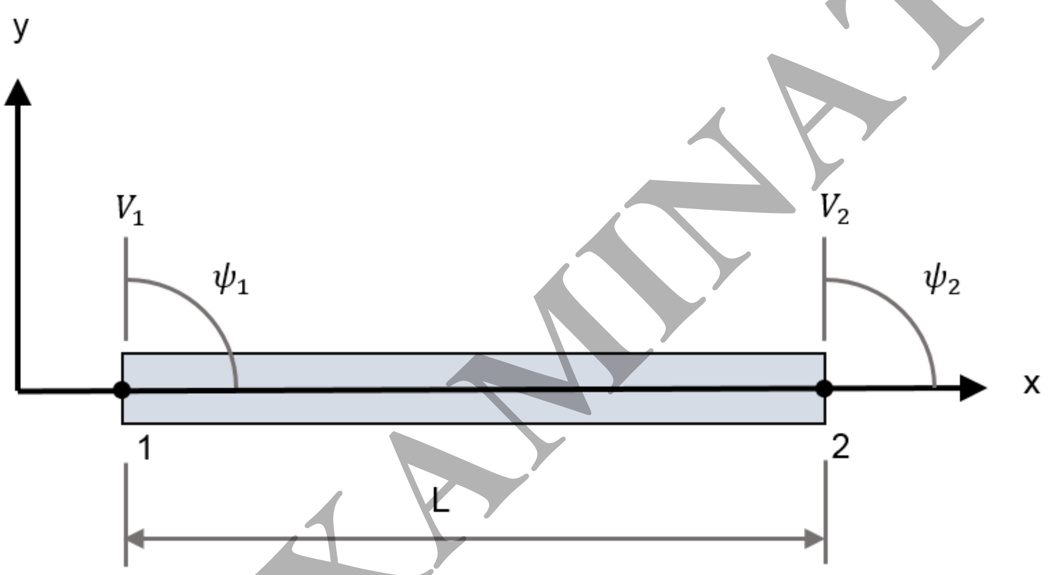


**FORMULAE SHEET**

**Dynamics**

$$(-\boldsymbol{\omega}^2\,[\boldsymbol{m}] + [\boldsymbol{K}])\,\{\boldsymbol{u}\} = \mathbf{0}$$

**Finite Element Notation for 2D Beams with 2 Nodes and 4 DOF:**




**Element Consistent Mass Matrix**

$$[m]^{\mathrm{e}} = \frac{\rho A L}{420}\begin{bmatrix} 156 & 22L & 54 & -13L \\ & 4L^2 & 13L & -3L^2 \\ & & 156 & -22L \\ & & & 4L^2 \end{bmatrix}$$

**Element Stiffness Matrix**

$$[K]^{\mathrm{e}} = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ & 4L^2 & -6L & 2L^2 \\ & & 12 & -6L \\ & & & 4L^2 \end{bmatrix}$$

**Element Displacement Functions**

$$v(x) = \left[1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3}, x - \frac{2x^2}{L} + \frac{x^3}{L^2}, \frac{3x^2}{L^2} - \frac{2x^3}{L^3}, -\frac{x^2}{L} + \frac{x^3}{L^2}\right]$$

**1- D Bar Element**

$$\boldsymbol{U(x) = \left(1 - \frac{x}{L}\right) U_1 + \left(\frac{x}{L}\right) U_2}$$

**1-D Beam Deflection Equation**

$$\left.\frac{dy}{dx}\right|_{x=x_i} = \frac{y_{i+1} - y_i}{\Delta x}$$

$$\left.\frac{dy}{dx}\right|_{x=x_i} = \frac{y_{i+1} - y_{i-1}}{2\Delta x}$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=x_i} = \frac{y_{i+1} - 2y_i + y_{i-1}}{\Delta x^2}$$

**1D steady-state heat diffusion equation** **(**applying the divergence theorem)

$$kA\frac{\partial^2 T}{\partial x^2} + (S * v) = 0$$

**END OF FORMULA SHEETS**

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