[ENG19]

UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

B.Eng. (Hons) AUTOMOTIVE PERFORMANCE ENGINEERING

SEMESTER ONE EXAMINATION 2022/23

ENGINEERING MATHEMATICS II

MODULE NO. MSP5017

Date: Tuesday 10th January 2023 **Time: 14.00-16.00**

INSTRUCTIONS TO CANDIDATES:

This is an open book examination.

There are FIVE questions.

Answer ALL FIVE questions.

The maximum marks possible foreach part is shown in brackets.

The examination is open-book.

The examination covers Learning Outcome 1. (See Module Handbook).

There is a formula sheet at the end of the paper.

School of Engineering
B.Eng. (Hons) Automotive Performance Engineering
Semester One Examination 2022/23
Engineering Mathematics II
Module Number: MSP5017

Question 1

Consider the following equation:

$$sin(t) - t + 1 = 0$$

a) Show that the interval $[1,2]$ contains a root of this equation.
(3 marks)

b) Use the Newton Raphson Method to find this root correct to 5 decimal places.
(12 marks)

Question 2

Consider the curve defined by the integral $\int_2^{10} \sqrt{2+x^2}dx$

a. Using Trapezoidal Rule, find the area under the curve using $n = 8$.
(9 marks)

b. Using Simpson's rule, find the area under the curve using $n = 8$.
(9 marks)

c. State which of the above you would consider to be the more accurate estimate and explain the reason for your answer.
(2 marks)

Question 3

The following Ordinary Differential Equation represents the quarter model for a car suspension system in the usual notation.

$$m\ddot{x} + c\dot{x} + kx = ky \qquad (1)$$

In what follows, assume that $m = 1, \quad c = 6 \quad, k = 10$ and that the car hits a step of height $y = 5$ at $t = 0$

The General Solution to (1) comprises the sum of a Complementary Function and a Particular Integral:

a) Find the Complementary Function. (5 marks)

b) Find the Particular Integral, and hence write down the General Solution (7 marks)

c) If the vertical displacement and velocity are zero at $t = 0$, write down the initial conditions, and use these to find the Particular Solution. (8 marks)

Question 4

Use the method of Laplace transforms to solve the following differential equations:

a) $\dot{x} + 3x = 6e^{3t} \qquad with \ \ x(0) = 0$ (7 marks)

b) $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0, \quad at\ x = 0,\ y = 0\ and\ \frac{dy}{dx} = 7$ (15 marks)

Question 5

Use the method of Laplace transforms to solve the differential equation below.

$$\frac{d^2P}{dt^2} + 3\frac{dP}{dt} + 2p = 4t \qquad p'(0) = p(0) = 0$$

(18 marks)

a. State if the differential equation is homogeneous or non-homogenous and explain the reason for your answer. (2 marks)

b. State if the system described by the differential equation is underdamped, critically damped or overdamped and explain the reason for your answer.

(3 marks)

END OF QUESTIONS

Formula sheets are over the page….

PLEASE TURN THE PAGE….

School of Engineering
B.Eng. (Hons) Automotive Performance Engineering
Semester One Examination 2022/23
Engineering Mathematics II
Module Number: MSP5017

FORMULA SHEET

Partial Fractions

proper fractions

$$\frac{f(x)}{(x+a)(x+b)(x+c)} = \frac{A}{(x+a)} + \frac{B}{(x+b)} + \frac{C}{(x+c)}$$

$$\frac{f(x)}{(x+a)^2(x+b)} = \frac{A}{(x+a)^2} + \frac{B}{(x+a)} + \frac{C}{(x+b)}$$

$$\frac{f(x)}{(x^2+a)(x+b)} = \frac{Ax+B}{(x^2+a)} + \frac{C}{(x+b)}$$

improper fractions — add on a polynomial of degree $n-d$
where n is the degree of the numerator
and d is the degree of the denominator

Quadratic Equation

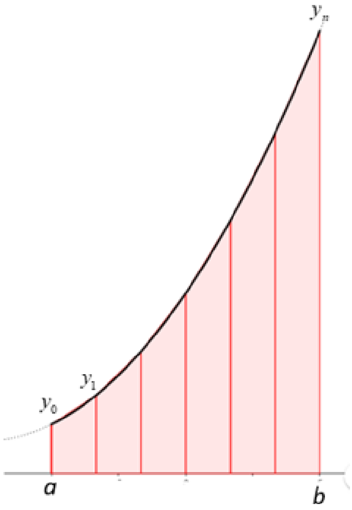
the solution to $ax^2 + bx + c = 0$

is $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

PLEASE TURN THE PAGE….

School of Engineering
B.Eng. (Hons) Automotive Performance Engineering
Semester One Examination 2022/23
Engineering Mathematics II
Module Number: MSP5017

The Trapezium rule



$$\int_a^b f(x)\,dx = T_n = \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

where $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$.

$$\text{Area} = \int_a^b y\,dx \approx \frac{1}{2}h\left[y_0 + 2(y_1 + y_2 + \ldots + y_{n-1}) + y_n\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Simpson's Rule

$$\text{Area} = \int_a^b f(x)\,dx$$

$$\approx \frac{\Delta x}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \ \ldots + 4y_{n-1} + y_n)$$

where $\Delta x = \dfrac{b-a}{n}$

Derivatives

[in all cases a is a constant]

Function $f(x)$	**Derivative** $\frac{d}{dx}f(x)$	
x^n	nx^{n-1}	
e^{ax}	ae^{ax}	
$\sin ax$	$a\cos ax$	
$\cos ax$	$-a\sin ax$	
$\tan ax$	$a\sec^2 ax$	
$\ln x$	$\frac{1}{x}$	
$\ln(ax+b)$	$\frac{a}{(ax+b)}$	
$\ln[u(x)]$	$\frac{1}{u}\frac{du}{dx}$	If $u(x)>0$
$a\sinh x$	$a\cosh ax$	
$a\cosh ax$	$a\sinh ax$	
$\tanh ax$	$a\left(1-\tanh^2 ax\right)$	

School of Engineering
B.Eng. (Hons) Automotive Performance Engineering
Semester One Examination 2022/23
Engineering Mathematics II
Module Number: MSP5017

Integrals

[in all cases a is a constant, and the constants of integration have been omitted]

Function $f(x)$	Integral $\int f(x)\,dx$	
x^n	$\frac{1}{n+1}x^{n+1}$	$n \neq -1$
$\frac{1}{x}$	$\ln x$	
e^{ax}	$\frac{1}{a}e^{ax}$	
$\sin ax$	$-\frac{1}{a}\cos ax$	
$\cos ax$	$\frac{1}{a}\sin ax$	
$\tan x$	$-\ln(\cos x)$	
$\ln(ax)$	$x\ln(ax) - x$	
$\frac{1}{(ax+b)}$	$\frac{1}{a}\ln(ax+b)$	
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(x/a\right)$	$a > x$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\left(x/a\right)$	
$\sinh x$	$\cosh x$	
$\cosh x$	$\sinh x$	

PLEASE TURN THE PAGE….

School of Engineering
B.Eng. (Hons) Automotive Performance Engineering
Semester One Examination 2022/23
Engineering Mathematics II
Module Number: MSP5017

Calculus Rules – Differentiation

product rule : $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule : $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{1}{v^2}\left[v\frac{du}{dx} - u\frac{dv}{dx}\right]$

chain rule : $\frac{d}{dx}\left[y\left(u(x)\right)\right] = \frac{dy}{du}\frac{du}{dx}$

Calculus Rules – Integration

integration by parts : $\int u\, dv = uv - \int v\, du$

or : $\int u\frac{dv}{dx}\, dx = uv - \int v\frac{du}{dx}dx$

with limits : $\int_a^b u\frac{dv}{dx}\, dx = \left[uv\right]_a^b - \int_a^b v\frac{du}{dx}\, dx$

integration by substitution : $\int f(u)\,\frac{du}{dx}\, dx = \int f(u)\, du$

for expressions in the form

$${}_a^b\int k\left[f(t)\right] f'(t)dt$$

Use the substitution $u = f(t)$

PLEASE TURN THE PAGE….

School of Engineering
B.Eng. (Hons) Automotive Performance Engineering
Semester One Examination 2022/23
Engineering Mathematics II
Module Number: MSP5017

2nd order Differential Equations

The differential equation $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$ [a, b, c constant]

has auxiliary equation $am^2 + bm + c = 0$ with solutions m_1 and m_2
and solutions:

(i) $y = Ae^{m_1x} + Be^{m_2x}$ if m_1 and m_2 are real and different

(ii) $y = (Ax + B)e^{mx}$ if m_1 and m_2 are real and equal

(iii) $y = e^{px}(A\cos qx + B\sin qx)$ if m_1 and m_2 are complex,

where $m_1 = p + jq$ and $m_2 = p - jq$

PLEASE TURN THE PAGE….

School of Engineering
B.Eng. (Hons) Automotive Performance Engineering
Semester One Examination 2022/23
Engineering Mathematics II
Module Number: MSP5017

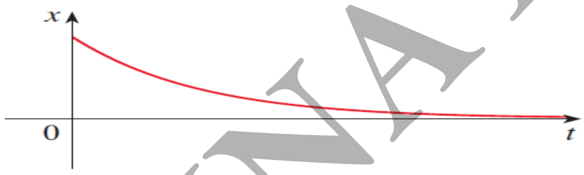
Damping

$$m\lambda^2 + c\lambda + k = 0$$

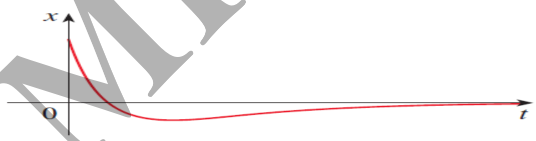
$$\lambda_1 = \frac{-c + \sqrt{c^2 - 4km}}{2m}$$

$$\lambda_1 = \frac{-c - \sqrt{c^2 - 4km}}{2m}$$

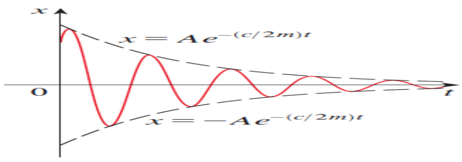
CASE I ■ $c^2 - 4mk > 0$ (overdamping)



CASE I ■ $c^2 - 4mk = 0$ (critical overdamping)



CASE III ■ $c^2 - 4mk < 0$ (underdamping)



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School of Engineering
B.Eng. (Hons) Automotive Performance Engineering
Semester One Examination 2022/23
Engineering Mathematics II
Module Number: MSP5017

Laplace Transforms

$$\mathcal{L}\left\{f(t)\right\} = \int_0^\infty e^{-st} f(t)\, dt$$

	Function $f(t)$	Laplace $\mathcal{L}\left\{f(t)\right\}$	ROC
1.	$a \quad (= \text{constant})$	$\frac{a}{s}$	$\Re(s) > 0$
2.	t	$\frac{1}{s^2}$	$\Re(s) > 0$
3.	t^2	$\frac{2}{s^3}$	$\Re(s) > 0$
4.	$t^n \quad \begin{bmatrix} n > 0 \\ \& \ n \in \mathbb{N} \end{bmatrix}$	$\frac{n!}{s^{n+1}}$	$\Re(s) > 0$
5.	e^{-at}	$\frac{1}{s+a}$	$\Re(s) > -a$
6.	$e^{-at}t^n$	$\frac{n!}{(s+a)^{n+1}}$	$\Re(s) > -a$
7.	$u(t-a)$	$\frac{e^{-as}}{s}$	$\Re(s) > 0$
8.	$y(t-a)u(t-a)$	$e^{-as}Y(s)$	where $Y(s) = \mathcal{L}\left\{y(t)\right\}$
9.	$\delta(t)$	1	$\forall s$
10.	$\delta(t-a)$	e^{-as}	$\forall s$

Laplace transforms continued

	Function	Laplace	ROC
11.	$\cos \omega t$	$\dfrac{s}{s^2+\omega^2}$	$\Re(s) > 0$
12.	$\sin \omega t$	$\dfrac{\omega}{s^2+\omega^2}$	$\Re(s) > 0$
13.	$e^{-at}\cos \omega t$	$\dfrac{s+a}{(s+a)^2+\omega^2}$	$\Re(s) > -a$
14.	$e^{-at}\sin \omega t$	$\dfrac{\omega}{(s+a)^2+\omega^2}$	$\Re(s) > -a$
15.	$\cosh \omega t$	$\dfrac{s}{s^2-\omega^2}$	
16.	$\sinh \omega t$	$\dfrac{\omega}{s^2-\omega^2}$	
17.	$e^{-at}\cosh \omega t$	$\dfrac{s+a}{(s+a)^2-\omega^2}$	
18.	$e^{-at}\sinh \omega t$	$\dfrac{\omega}{(s+a)^2-\omega^2}$	
19.	$\dfrac{d}{dt}\{y(t)\}$	$sY(s)-y_0$	where $Y(s)=\mathcal{L}\{y(t)\}$ and $y_0 = y(0)$
20.	$\dfrac{d^2}{dt^2}\{y(t)\}$	$s^2Y(s)-sy_0-\dot{y}_0$	where $\dot{y}_0 = \left.\dfrac{dy}{dt}\right\vert_{t=0}$

END OF PAPER