[ENG18]

UNIVERSITY OF BOLTON

NATIONAL CENTRE FOR MOTORSPORT ENGINEERING

BEng (HONS) AUTOMOTIVE PERFORMANCE ENGINEERING (MOTORSPORT)

SEMESTER ONE EXAMINATION 2022/2023

ENGINEERING MATHEMATICS

MODULE NO. MSP4017

Date Tuesday 10th January 2023 **Time: 14.00-16.00**

INSTRUCTIONS TO CANDIDATES:

This is an open book examination

This paper has FIVE questions.

Answer ALL FIVE questions.

There is a formula sheet at the end of the paper.

The maximum marks possible for each question are shown in brackets.

Electronic calculators may be used if data and program storage memory is cleared prior to the examination.

Mobile phones or tablets may-not be used as calculators.

Question 1

a) If $A = \begin{pmatrix} 2 & -5 \\ 8 & -3 \end{pmatrix}$ and $B = \begin{pmatrix} -3 & 5 \\ -8 & 2 \end{pmatrix}$ find:

(i) $A + 2B$. (4 marks)

(ii) AB and BA. (5 marks)

(ii) $detA$; (3 marks)

(iii) A^{-1}. (3 marks)

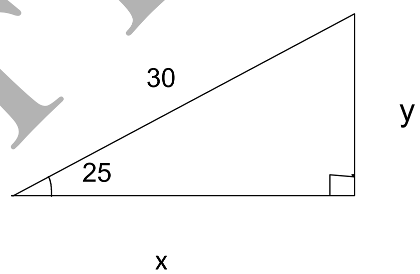
b) Find the inverse of the following matrix:

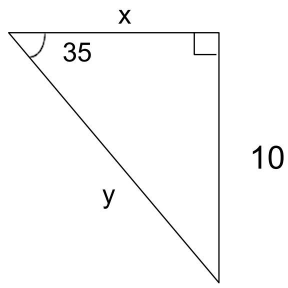
$$\begin{bmatrix} 2 & 3 & 5 \\ 4 & 1 & 4 \\ 5 & 8 & 1 \end{bmatrix}$$

(10 marks)

Question 2

(a) For each of the right-angled triangles below, use trigonometry to calculate the lengths *x* and *y* of the sides indicated:





(10 marks)

Question 2 continues over the page….

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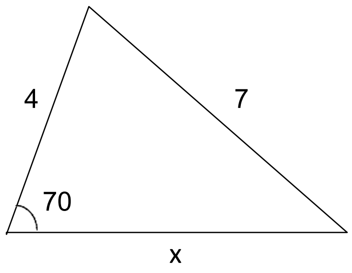
Question 2 continued….

(b) Find the length *x* of the side indicated in the following triangle:



(10 marks)

Question 3

a) Differentiate $y = 2x^2$ from first principles: (4 marks)

b) Calculate the derivative of the following functions:

(i) $y = x^2 sin4x$ (4 marks)

(ii) $y = \cos(x^2)$ (3 marks)

(iii) $y = \frac{\sin(2x)}{x^2-7}$ (4 marks)

c) Determine the stationary point(s) of the function

$$y = f(x) = x^3 - 6x^2 - 63x + 14$$

and also determine if they are local maxima or minima.

(10 marks)

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Question 4

Calculate the following integrals:

(i) $\int sin2x \ + cos3x \ .dx$ (3 marks)

(ii) $\int e^{-3x}.dx$ (2 marks)

(iii) $\int_1^3 (2x^2 - 3x + 2).dx$ (5 marks)

Question 5

a) (i) Express $z = 4 + 7i$ in polar form. (4 marks)

(ii) Simplify $(-1 + i)^2$. (4 marks)

b) Given $z = \frac{2+i}{1-i}$, find the real and complex parts of:

(i) z (5 marks)

(ii) $z + z^2$ (4 marks)

c) If $z_1 = 3\angle 225^0$ $\quad z_2 = 18\angle 175^0$

find the value of $\frac{z_1}{z_2}$ (3 marks)

END OF QUESTIONS

Formula sheet over the page….

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FORMULA SHEET

Quadratic Equation

the solution to $ax^2 + bx + c = 0$

is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Derivatives

[in all cases a is a constant]

Function $f(x)$	**Derivative** $\frac{d}{dx}f(x)$	
x^n	nx^{n-1}	
e^{ax}	ae^{ax}	
$\sin ax$	$a\cos ax$	
$\cos ax$	$-a\sin ax$	
$\tan ax$	$a\sec^2 ax$	
$\ln x$	$\frac{1}{x}$	
$\ln(ax+b)$	$\frac{a}{(ax+b)}$	
$\ln[u(x)]$	$\frac{1}{u}\frac{du}{dx}$	If $u(x)>0$
$a\sinh x$	$a\cosh ax$	
$a\cosh ax$	$a\sinh ax$	
$\tanh ax$	$a(1-\tanh^2 ax)$	

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Integrals

[in all cases a is a constant, and the constants of integration have been omitted]

Function $f(x)$	**Integral** $\int f(x)\,dx$	
x^n	$\frac{1}{n+1}x^{n+1}$	$n \neq -1$
$\frac{1}{x}$	$\ln x$	
e^{ax}	$\frac{1}{a}e^{ax}$	
$\sin ax$	$-\frac{1}{a}\cos ax$	
$\cos ax$	$\frac{1}{a}\sin ax$	
$\tan x$	$-\ln(\cos x)$	
$\ln(ax)$	$x\ln(ax)-x$	
$\frac{1}{(ax+b)}$	$\frac{1}{a}\ln(ax+b)$	
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(x/a\right)$	$a > x$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\left(x/a\right)$	
$\sinh x$	$\cosh x$	
$\cosh x$	$\sinh x$	

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Calculus Rules – Differentiation

product rule : $$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

quotient rule : $$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{1}{v^2}\left[v\frac{du}{dx} - u\frac{dv}{dx}\right]$$

chain rule : $$\frac{d}{dx}\left[y\left(u(x)\right)\right] = \frac{dy}{du}\frac{du}{dx}$$

Parametric Differentiation

First derivative: $$\frac{dy}{dx} = \frac{dt}{dx}\cdot\frac{dy}{dt} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

Second derivative: $$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

Calculus Rules – Integration

integration by parts : $$\int u\,dv = uv - \int v\,du$$

or : $$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

with limits : $$\int_a^b u\frac{dv}{dx}\,dx = \left[uv\right]_a^b - \int_a^b v\frac{du}{dx}\,dx$$

integration by substitution : $$\int f(u)\frac{du}{dx}\,dx = \int f(u)\,du$$

for expressions in the form

$$\int_a^b k\left[f(t)\right] f'(t)\,dt$$

Use the substitution $u = f(t)$

Sine Rule

$$\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$$

Cosine rule

$$a^2 = b^2 + c^2 - 2bc\cos(A) \quad or \quad \cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$$

$$b^2 = a^2 + c^2 - 2ac\cos(B) \quad or \quad \cos(B) = \frac{a^2 + c^2 - b^2}{2ac}$$

$$c^2 = b^2 + a^2 - 2bc\cos(C) \quad or \quad \cos(C) = \frac{a^2 + b^2 - c^2}{2ab}$$

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Complex numbers

$$j = \sqrt{-1}$$

if $z = a + jb$ then $\Re(z) = a$

$\Im(z) = b$

$r = |z| = \mathrm{mod}(z) = \sqrt{a^2 + b^2}$

$\theta = \arg(z) = \tan^{-1}\left(\frac{b}{a}\right)$

and $z^* = a - jb$

cartesian form $z = a + jb$

polar form $z = r\angle\theta$ (express θ in degrees)

exponential form $z = re^{j\theta}$ (express θ in radians)

trigonometric form $z = r(\cos\theta + j\sin\theta)$

Matrices

Determination of a 2x2 matrix:

$$\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - cb$$

Inverse of a 2x2 matrix

$$A^{-1} = \frac{1}{ad - cd}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \text{where } \det(\mathrm{A}) \neq 0$$

Determination of a 3x3 matrix:

Inverse of any square matrix is given by:

$$A^{-1} = \frac{adjA}{\det A}$$

$$adjA = C^T$$

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$$\text{if matrix A} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$\det A = a\left[\det\begin{pmatrix} e & f \\ h & i \end{pmatrix}\right] - b\left[\det\begin{pmatrix} d & f \\ g & i \end{pmatrix}\right] + c\left[\det\begin{pmatrix} d & e \\ g & h \end{pmatrix}\right]$$

END OF PAPER