

UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

MSC SYSTEM ENGINEERING AND MANAGEMENT

SEMESTER ONE EXAMINATION 2021/2022

SIGNAL PROCESSING

MODULE NO: EEM7011

Date: Wednesday 12th January 2022 Time: 14:00 – 16:00

INSTRUCTIONS TO CANDIDATES:

There are **SIX** questions.

Answer **FOUR** questions.

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

Useful Transform Tables have been provided at the end of this examination paper.

This examination paper carries a total of 100 marks.

All working must be shown. A numerical solution to a question obtained by programming an electronic calculator will not be accepted.

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Question 1

- a) Using suitable illustrative examples, one for each, differentiate between odd and even signals. **[6 marks]**
- b) The signal depicted in Figure Q1b is represented as $g(t)$, after transformation, the resulting transformed signal is given as $h(t) = 1.5g(t) - 1$,

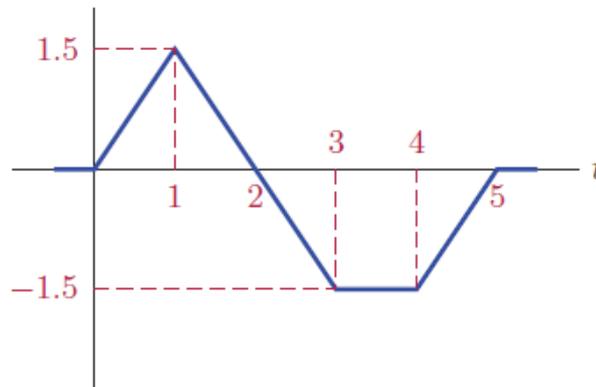


Figure Q1b

- i) Identify the type of transformation undergone by signal **[2 marks]**
- ii) Draw suitable transformation table for the signal **[4 marks]**
- iii) Draw the resulting signal $h(t)$. **[4 marks]**
- c) Determine and plot the following signal $f[n] = u[n] + (1 - n)u[n - 1] + (n - 2)u[n - 2]$, where

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & \text{elsewhere} \end{cases}$$

and

$$-2 \leq n \leq 4.$$

[9 marks]

Total 25 marks

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Question 2

A continuous signal described by $x(t) = \cos(2000\pi t)$ is sampled at a sampling rate of 1900 Hz.

- i. determine the frequency of the continuous signal **[5 marks]**
- ii. determine the frequency of the digitised signal. **[10 marks]**
- iii. illustrate whether the signal is even or odd signal **[4 marks]**
- iv. determine the apparent frequency at the 1st, 3rd and 5th sample (i.e. $n = 1, 3, 5$). **[6 marks]**

Total 25 marks

Question 3

a) If the double-sided z-transform of a digital signal $x[n]$ is:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

Consider a digital signal represented as $x[n] = na^n u[n]$,

- i. determine the z-transform of the signal **[5 marks]**
- ii. if $a = 1$, determine the region of convergence of the signal. **[5 marks]**

Note that,

$$\frac{d}{dz}X(z) = \frac{d}{dz} \left[\sum_{n=-\infty}^{\infty} x[n]z^{-n} \right].$$

b) Define a digital signal as $f[n] = x[n] + 0.5x[n-1] + x[n-2]$,

- i. determine the z-transform of the signal **[5 marks]**
- ii. determine the transfer function of the equivalent system **[5 marks]**
- iii. using direct method, calculate the frequency response of the signal. **[5 marks]**

Total 25 marks

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Question 4

a) Briefly discuss the conditions necessary for a digital filter to have a linear phase characteristic and the advantage of filters with such characteristics. **[5 marks]**

b) An FIR filter has its impulse response, $h[n]$ defined over $0 \leq n \leq N - 1$ interval. Show that if $N = 8$ and $h[n]$ satisfies the following symmetry condition:

$$h[n] = h[N - 1 - n]$$

the phase of the filter is linear in nature whose generic value is given by

$$\angle H(e^{j\omega}) = \frac{N - 1}{2} \omega.$$

[20 marks]

Total 25 marks

Question 5

a) Z-transform and Laplace transform are useful tools in signal processing. Considering an LTI system,

i) Describe the difference between Z-transform and Laplace transform. **[2 marks]**

ii) Using suitable illustrative examples, explain the relationships between z-plane and s-plane. **[6 marks]**

b) If the transfer function of the LTI system is given by

$$H(s) = \frac{s + 2}{s^2 - 5s + 6}$$

i) Determine the poles and zeros of the system. **[3 marks]**

ii) The LTI system is being considered for digital filter operation and requires that it should be converted, using impulse invariant method, determine the equivalent digital filter, if the sampling frequency is 8 Hz. **[14 marks]**

Total 25 marks

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Question 6

- a) Filters are useful tools in engineering and non-engineering industries. They may be analogue or digital filters. Using suitable representative models, where necessary, differentiate between analogue and digital filters. **[6 marks]**
- b) Explain any five advantages of digital filters over analogue filters. **[5 marks]**
- c) Recursive filters may be referred to as infinite impulse response filters. The IIR filters form an effective way of achieving long impulse response without performing a long division. An example of an analogue form of such filters in s-domain is:

$$X(s) = \frac{s^2 + 1}{s^2 + 5s + 6}$$

Determine $x(t)$. Note that $\mathcal{L}^{-1}\left\{\frac{1}{s+\alpha}\right\} = e^{-\alpha t}u(t)$. **[14 marks]**

Total 25 marks

END OF QUESTIONS

Useful Transform Tables follow on the next page....

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$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1. 1	$\frac{1}{s}$
2. e^{at}	$\frac{1}{s-a}$
3. t^n , $n = \text{positive integer}$	$\frac{n!}{s^{n+1}}$
4. t^p , $p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5. $\sin at$	$\frac{a}{s^2 + a^2}$
6. $\cos at$	$\frac{s}{s^2 + a^2}$
7. $\sinh at$	$\frac{a}{s^2 - a^2}$
8. $\cosh at$	$\frac{s}{s^2 - a^2}$
9. $e^{at} \sin bt$	$\frac{b}{(s-a)^2 + b^2}$
10. $e^{at} \cos bt$	$\frac{s-a}{(s-a)^2 + b^2}$
11. $t^n e^{at}$, $n = \text{positive integer}$	$\frac{n!}{(s-a)^{n+1}}$
12. $u_c(t)$	$\frac{e^{-cs}}{s}$
13. $u_c(t)f(t-c)$	$e^{-cs}F(s)$
14. $e^{ct}f(t)$	$F(s-c)$
15. $f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$
16. $(f * g)(t) = \int_0^t f(t-\tau)g(\tau) d\tau$	$F(s)G(s)$
17. $\delta(t-c)$	e^{-cs}
18. $f^{(n)}(t)$	$s^n F(s) - s^{n-1}f(0) - \dots - f^{(n-1)}(0)$
19. $(-t)^n f(t)$	$F^{(n)}(s)$

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Signal $x[n]$	z -Transform $X(z)$	ROC
$\delta[n]$	1	All z
$u[n]$	$\frac{1}{1-z^{-1}}$	$ z > 1$
$n u[n]$	$\frac{z^{-1}}{(1-z^{-1})^2}$	$ z > 1$
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$ z > a $
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$ z > a $
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$ z < a $
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$ z < a $
$(\cos \omega_0 n)u[n]$	$\frac{1-z^{-1} \cos \omega_0}{1-2z^{-1} \cos \omega_0 + z^{-2}}$	$ z > 1$
$(\sin \omega_0 n)u[n]$	$\frac{z^{-1} \sin \omega_0}{1-2z^{-1} \cos \omega_0 + z^{-2}}$	$ z > 1$
$(a^n \cos \omega_0 n)u[n]$	$\frac{1-az^{-1} \cos \omega_0}{1-2az^{-1} \cos \omega_0 + a^2 z^{-2}}$	$ z > a $
$(a^n \sin \omega_0 n)u[n]$	$\frac{az^{-1} \sin \omega_0}{1-2az^{-1} \cos \omega_0 + a^2 z^{-2}}$	$ z > a $

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