[ENG17]

UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

BENG (HONS) ELECTRICAL & ELECTRONICS ENGINEERING

SEMESTER ONE EXAMINATION 2021/2022

ENGINEERING ELECTROMAGNETISM

MODULE NO: EEE6012

Date: Friday 14th January 2022 Time: 10:00 – 12:00

INSTRUCTIONS TO CANDIDATES:	**There are SIX questions.** **Answer ANY FOUR questions.** **All questions carry equal marks.** **Marks for parts of questions are shown in brackets.** **Electronic calculators may be used provided that data and program storage memory is cleared prior to the examination.**
CANDIDATES REQUIRE:	**Formula Sheet (attached).**

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Semester One Examination 2021/2022
Engineering Electromagnetism
Module Number: EEE6012

Question 1

Q1. a) An electromagnetic wave is propagating in the y-direction in a lossy medium with attenuation constant $\alpha = 0.5\frac{Np}{m}$. If the wave's electric-field amplitude is 100 V/m at y=0. How far can the wave travel before its amplitude will have been reduced to (i) 15 V/m , (ii) 1.5 V/m, (iii) $1\mu V/m$. **[5 marks]**

b) A series RLC circuit is connected to a voltage source given by $v_S(t) = 150\cos\omega t\ V$. Find (i) the phasor current I and (ii) the instantaneous current $i(t)$ for R=400 Ω, L= 3 mH, C=20 n$F, and\ \omega = 10^5 rad/s$. **[8 marks]**

c) Two points in a Cartesian coordinates are P1(1,2,3) and P2(-1,-2,3). Find

(i) the distance vector between P1 and P2.
(ii) the angle between vectors $\overrightarrow{OP1}$ and $\overrightarrow{OP2}$ using the cross product between them.
(iii) the angle between vector $\overrightarrow{OP2}$ and the y-axis.

[12 marks]

Total 25 marks

Question 2

a) a) Transform vector **A**=$\hat{\boldsymbol{x}}\,(x+y) + \hat{\boldsymbol{y}}\,(y-x) + \hat{\boldsymbol{z}}\,z$ from Cartesian to Cylindrical coordinates. **[6 marks]**

b) A scalar quantity of $V = rz^2 cos2\phi$. Find its directional derivative along the direction **A**=$\hat{\boldsymbol{r}}\,\mathbf{2} - \hat{\boldsymbol{z}}\mathbf{3}$ and evaluate it at $(1,0.5\pi,2)$. **[9 marks]**

c) Find the divergence and the curl of the given vector A=$e^{-7y}(\hat{\boldsymbol{x}}\,sin3x$+$\hat{\boldsymbol{y}}$cos3x) at x=10 and y=1.0. **[10 marks]**

Total 25 marks

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Question 3

a) A circular disk sitting at the x-y plane with z=0 is characterised by an azimuthally symmetric surface charge density that increases linearly with radius r from zero to 8 C/m^2 at r=4 cm. Find the total charge present on the disk surface.

[6 marks]

b) Two point charges with q1=3X10^{-5} C and q2=-5X10^{-5} C are located in free space at points with Cartesian coordinates (1,3,-1) and (-3,1,-2) respectively . Find

i) the electric field **E** at (3,1,-2) **[7 marks]**

ii) the force on a 7X10^{-5} C charge located at that point. All distances are in metres. **[3 marks]**

c) Derive a formula for the inductance of a cylindrical conductor due to (i) internal flux and (ii) external flux assuming the conductor is surrounded by air.

[9 marks]

Total 25 marks

Question 4

a) An Ethernet cable has L= 0.22 μHm^{-1} and C = 86 pFm^{-1}. What is the wavelength at 10 MHz ? **[6 marks]**

b) Assuming the loss resistance of a half-wave dipole antenna to be negligibly small and ignoring the reactance component of its antenna impedance, calculate the standing wave ratio on a 50-W transmission line connected to the dipole antenna. Note that a half wave dipole has a radiation resistance of 73 Ω.

[6 marks]

c) A transponder with a bandwidth of 400 MHz uses polarization diversity. If the bandwidth allocated to transmit a single telephone channel is 4 kHz, how many telephone channels can be carried by the transponder? **[4 marks]**

d) A remote sensing satellite is in circular orbit around the earth at an altitude of 1,100 km above the earth's surface. What is its orbital period? Assume that the radius of the earth is 6,378 km. **[9 marks]**

Total 25 marks

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Question 5

A source $[\tilde{V}_g = 100\angle 0°\ V, Z_g = R_g = 50\ \Omega, f = 100\ MHz]$ is connected to a lossless transmission line [L = 0.25μH/m, C=100pF/m, l=10m]. For loads of $Z_L = R_L$ = 100 Ω, determine the

a) reflection coefficient at the load **[6 marks]**

b) standing wave ratio **[2 marks]**

c) input impedance at the transmission line input terminals **[7 marks]**

d) voltage along the transmission line for load of $Z_L = R_L =$ 100 Ω. **[10 marks]**

Total 25 marks

Question 6

a) An antenna has a field pattern given by $E(\theta) = cos\theta cos2\theta$ for $0° \leq \theta \leq 90°$. Calculate the:

   i. half-power beamwidth (HPBW) **[7 marks]**

   ii. beamwidth between first nulls (FNBW) **[3 marks]**

b) A lossless resonant half-wavelength dipole antenna, with input impedance of 73 Ω, is connected to a transmission line whose characteristic impedance is 50 ohms. Assuming that the pattern of the antenna is given by approximately $U = B_0 \sin^3 \theta$. Determine the maximum dB gain of this antenna. **[15 marks]**

Total 25 marks

END OF QUESTIONS

Formula sheet follows over the page….

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Formula sheet

These equations are given to save short-term memorisation of details of derived equations and are given without any explanation or definition of symbols; the student is expected to know the meanings and usage.

Time-domain sinusoidal functions $z(t)$ and their cosine-reference phasor-domain counterparts $\widetilde{Z}$, where $z(t) = \mathfrak{Re}\ [\widetilde{Z}e^{j\omega t}]$.

$z(t)$		$\widetilde{Z}$
$A\cos\omega t$	$\leftrightarrow$	A
$A\cos(\omega t + \phi_0)$	$\leftrightarrow$	$Ae^{j\phi_0}$
$A\cos(\omega t + \beta x + \phi_0)$	$\leftrightarrow$	$Ae^{j(\beta x+\phi_0)}$
$Ae^{-\alpha x}\cos(\omega t + \beta x + \phi_0)$	$\leftrightarrow$	$Ae^{-\alpha x}e^{j(\beta x+\phi_0)}$
$A\sin\omega t$	$\leftrightarrow$	$Ae^{-j\pi/2}$
$A\sin(\omega t + \phi_0)$	$\leftrightarrow$	$Ae^{j(\phi_0-\pi/2)}$
$\frac{d}{dt}(z(t))$	$\leftrightarrow$	$j\omega\widetilde{Z}$
$\frac{d}{dt}[A\cos(\omega t + \phi_0)]$	$\leftrightarrow$	$j\omega Ae^{j\phi_0}$
$\int z(t)\,dt$	$\leftrightarrow$	$\frac{1}{j\omega}\widetilde{Z}$
$\int A\sin(\omega t + \phi_0)\,dt$	$\leftrightarrow$	$\frac{1}{j\omega}Ae^{j(\phi_0-\pi/2)}$

Summary of vector relations.

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Coordinate variables	x, y, z	r, ϕ, z	R, θ, ϕ
Vector representation $\mathbf{A} =$	$\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{r}}A_r + \hat{\boldsymbol{\phi}}A_\phi + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{R}}A_R + \hat{\boldsymbol{\theta}}A_\theta + \hat{\boldsymbol{\phi}}A_\phi$
Magnitude of A $\lvert\mathbf{A}\rvert =$	$\sqrt[+]{A_x^2 + A_y^2 + A_z^2}$	$\sqrt[+]{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt[+]{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector $\overrightarrow{OP_1} =$	$\hat{\mathbf{x}}x_1 + \hat{\mathbf{y}}y_1 + \hat{\mathbf{z}}z_1$, for $P = (x_1, y_1, z_1)$	$\hat{\mathbf{r}}r_1 + \hat{\mathbf{z}}z_1$, for $P = (r_1, \phi_1, z_1)$	$\hat{\mathbf{R}}R_1$, for $P = (R_1, \theta_1, \phi_1)$
Base vectors properties	$\hat{\mathbf{x}} \cdot \hat{\mathbf{x}} = \hat{\mathbf{y}} \cdot \hat{\mathbf{y}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}} \cdot \hat{\mathbf{y}} = \hat{\mathbf{y}} \cdot \hat{\mathbf{z}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}} \times \hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}} \times \hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}} \times \hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}} \cdot \hat{\mathbf{r}} = \hat{\boldsymbol{\phi}} \cdot \hat{\boldsymbol{\phi}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}} \cdot \hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}} \cdot \hat{\mathbf{z}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}} \times \hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\phi}} \times \hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}} \times \hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}$	$\hat{\mathbf{R}} \cdot \hat{\mathbf{R}} = \hat{\boldsymbol{\theta}} \cdot \hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}} \cdot \hat{\boldsymbol{\phi}} = 1$ $\hat{\mathbf{R}} \cdot \hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}} \cdot \hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}} \cdot \hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}} \times \hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}$ $\hat{\boldsymbol{\theta}} \times \hat{\boldsymbol{\phi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\phi}} \times \hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product $\mathbf{A} \cdot \mathbf{B} =$	$A_x B_x + A_y B_y + A_z B_z$	$A_r B_r + A_\phi B_\phi + A_z B_z$	$A_R B_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product $\mathbf{A} \times \mathbf{B} =$	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\phi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length $d\mathbf{l} =$	$\hat{\mathbf{x}}\, dx + \hat{\mathbf{y}}\, dy + \hat{\mathbf{z}}\, dz$	$\hat{\mathbf{r}}\, dr + \hat{\boldsymbol{\phi}} r\, d\phi + \hat{\mathbf{z}}\, dz$	$\hat{\mathbf{R}}\, dR + \hat{\boldsymbol{\theta}} R\, d\theta + \hat{\boldsymbol{\phi}} R \sin\theta\, d\phi$
Differential surface areas	$d\mathbf{s}_x = \hat{\mathbf{x}}\, dy\, dz$ $d\mathbf{s}_y = \hat{\mathbf{y}}\, dx\, dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}\, dx\, dy$	$d\mathbf{s}_r = \hat{\mathbf{r}} r\, d\phi\, dz$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}\, dr\, dz$ $d\mathbf{s}_z = \hat{\mathbf{z}} r\, dr\, d\phi$	$d\mathbf{s}_R = \hat{\mathbf{R}} R^2 \sin\theta\, d\theta\, d\phi$ $d\mathbf{s}_\theta = \hat{\boldsymbol{\theta}} R \sin\theta\, dR\, d\phi$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}} R\, dR\, d\theta$
Differential volume $dV =$	$dx\, dy\, dz$	$r\, dr\, d\phi\, dz$	$R^2 \sin\theta\, dR\, d\theta\, d\phi$

Coordinate transformation relations.

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to cylindrical	$r = \sqrt[+]{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{\mathbf{r}} = \hat{\mathbf{x}} \cos\phi + \hat{\mathbf{y}} \sin\phi$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}} \sin\phi + \hat{\mathbf{y}} \cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_r = A_x \cos\phi + A_y \sin\phi$ $A_\phi = -A_x \sin\phi + A_y \cos\phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{\mathbf{x}} = \hat{\mathbf{r}} \cos\phi - \hat{\boldsymbol{\phi}} \sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{r}} \sin\phi + \hat{\boldsymbol{\phi}} \cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_x = A_r \cos\phi - A_\phi \sin\phi$ $A_y = A_r \sin\phi + A_\phi \cos\phi$ $A_z = A_z$
Cartesian to spherical	$R = \sqrt[+]{x^2 + y^2 + z^2}$ $\theta = \tan^{-1}[\sqrt[+]{x^2 + y^2}/z]$ $\phi = \tan^{-1}(y/x)$	$\hat{\mathbf{R}} = \hat{\mathbf{x}} \sin\theta\cos\phi + \hat{\mathbf{y}} \sin\theta\sin\phi + \hat{\mathbf{z}} \cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{x}} \cos\theta\cos\phi + \hat{\mathbf{y}} \cos\theta\sin\phi - \hat{\mathbf{z}} \sin\theta$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}} \sin\phi + \hat{\mathbf{y}} \cos\phi$	$A_R = A_x \sin\theta\cos\phi + A_y \sin\theta\sin\phi + A_z \cos\theta$ $A_\theta = A_x \cos\theta\cos\phi + A_y \cos\theta\sin\phi - A_z \sin\theta$ $A_\phi = -A_x \sin\phi + A_y \cos\phi$
Spherical to Cartesian	$x = R\sin\theta\cos\phi$ $y = R\sin\theta\sin\phi$ $z = R\cos\theta$	$\hat{\mathbf{x}} = \hat{\mathbf{R}} \sin\theta\cos\phi + \hat{\boldsymbol{\theta}} \cos\theta\cos\phi - \hat{\boldsymbol{\phi}} \sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{R}} \sin\theta\sin\phi + \hat{\boldsymbol{\theta}} \cos\theta\sin\phi + \hat{\boldsymbol{\phi}} \cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{R}} \cos\theta - \hat{\boldsymbol{\theta}} \sin\theta$	$A_x = A_R \sin\theta\cos\phi + A_\theta \cos\theta\cos\phi - A_\phi \sin\phi$ $A_y = A_R \sin\theta\sin\phi + A_\theta \cos\theta\sin\phi + A_\phi \cos\phi$ $A_z = A_R \cos\theta - A_\theta \sin\theta$
Cylindrical to spherical	$R = \sqrt[+]{r^2 + z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{\mathbf{R}} = \hat{\mathbf{r}} \sin\theta + \hat{\mathbf{z}} \cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{r}} \cos\theta - \hat{\mathbf{z}} \sin\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$	$A_R = A_r \sin\theta + A_z \cos\theta$ $A_\theta = A_r \cos\theta - A_z \sin\theta$ $A_\phi = A_\phi$
Spherical to cylindrical	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{\mathbf{r}} = \hat{\mathbf{R}} \sin\theta + \hat{\boldsymbol{\theta}} \cos\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$ $\hat{\mathbf{z}} = \hat{\mathbf{R}} \cos\theta - \hat{\boldsymbol{\theta}} \sin\theta$	$A_r = A_R \sin\theta + A_\theta \cos\theta$ $A_\phi = A_\phi$ $A_z = A_R \cos\theta - A_\theta \sin\theta$

ELECTROSTATICS:

$$\mathbf{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\mathbf{a}_{R_{12}}, \quad \mathbf{F} = \frac{Q}{4\pi\varepsilon_0}\sum_{k=1}^{N}\frac{Q_k(\mathbf{r}-\mathbf{r}_k)}{|\mathbf{r}-\mathbf{r}_k|^3}, \quad \mathbf{E} = \frac{\mathbf{F}}{Q}, \quad \mathbf{E} = \int \frac{\rho_L dl}{4\pi\varepsilon_0 R^2}\mathbf{a}_R, \quad \mathbf{E} = \int \frac{\rho_S dS}{4\pi\varepsilon_0 R^2}\mathbf{a}_R, \quad \mathbf{E} = \int \frac{\rho_v dv}{4\pi\varepsilon_0 R^2}\mathbf{a}_R$$

$$\mathbf{E} = \frac{\rho_S}{2\varepsilon_0}\mathbf{a}_n, \quad \mathbf{E} = \frac{\rho_L}{2\pi\varepsilon_0\rho}\mathbf{a}_\rho, \quad Q = \oint_S \mathbf{D}\cdot d\mathbf{S} = \int_v \rho_v dv, \quad \nabla\cdot\mathbf{D} = \rho_v, \quad W = -Q\int_A^B \mathbf{E}\cdot d\ell, \quad V_{AB} = \frac{W}{Q} = -\int_A^B \mathbf{E}\cdot d\ell, \quad V = \frac{Q}{4\pi\varepsilon_0 r}$$

$$\oint \mathbf{E}\cdot d\ell = 0, \quad \nabla\times\mathbf{E} = 0, \quad \mathbf{E} = -\nabla V, \quad W_E = \frac{1}{2}\sum_{k=1}^{n} Q_k V_k, \quad W_E = \frac{1}{2}\int \mathbf{D}\cdot\mathbf{E}dv = \frac{1}{2}\int \varepsilon_0 E^2 dv, \quad \mathbf{J} = \rho_v\mathbf{u}, \quad I = \int_S \mathbf{J}\cdot d\mathbf{S}, \quad \mathbf{J} = \sigma\mathbf{E},$$

$$R = \frac{\mathrm{V}}{\mathrm{I}} = \frac{\int \mathbf{E}\cdot d\mathbf{l}}{\int \sigma\mathbf{E}\cdot d\mathbf{S}}, \quad \mathbf{D} = \varepsilon\,\mathbf{E}, \quad \nabla\cdot\mathbf{J} = -\frac{\partial\rho_v}{\partial t}, \quad E_{1t} = E_{2t}, \quad D_{1n} - D_{2n} = \rho_S, \quad D_{1n} = D_{2n}, \quad \frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}$$

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon}, \quad \nabla^2 V = 0, \quad C = \frac{Q}{V} = \frac{\varepsilon\oint \mathbf{E}\cdot d\mathbf{S}}{\int \mathbf{E}\cdot d\mathbf{l}}, \quad W_E = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{Q^2}{2C}, \quad C = \frac{Q}{V} = \frac{2\pi\varepsilon L}{\ln\frac{b}{a}}, \quad C = \frac{Q}{V} = \frac{4\pi\varepsilon}{\frac{1}{a}-\frac{1}{b}}, \quad RC = \frac{\varepsilon}{\sigma}$$

$$\epsilon_o = 8.85X10^{-12}F/m \quad , \mu_o = 4\pi X10^{-7}\ H/m$$

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MAGNETOSTATICS:

$$\mathbf{H}=\int_L \frac{Id\mathbf{l}\times\mathbf{a}_R}{4\pi R^2},\ \mathbf{H}=\int_S \frac{\mathbf{K}dS\times\mathbf{a}_R}{4\pi R^2},\ \mathbf{H}=\int_v \frac{\mathbf{J}dv\times\mathbf{a}_R}{4\pi R^2},\ \mathbf{H}=\frac{I}{4\pi\rho}(\cos\alpha_2-\cos\alpha_1)\mathbf{a}_\phi,\ \mathbf{H}=\frac{I}{2\pi\rho}\mathbf{a}_\phi,\ \mathbf{a}_\phi=\mathbf{a}_\ell\times\mathbf{a}_\rho,$$

$$\oint \mathbf{H}\cdot d\mathbf{l}=I_{enc},\ \nabla\times\mathbf{H}=\mathbf{J},\ \mathbf{H}=\frac{I}{2\pi\rho}\mathbf{a}_\phi,\ \mathbf{H}=\frac{1}{2}\mathbf{K}\times\mathbf{a}_n,\ \mathbf{B}=\mu\mathbf{H},\ \Psi=\int_S \mathbf{B}\cdot d\mathbf{S},\ \oint \mathbf{B}\cdot d\mathbf{S}=0,\ \nabla\cdot\mathbf{B}=0,\ \mathbf{H}=-\nabla V_m,$$

$$\mathbf{B}=\nabla\times\mathbf{A},\ \mathbf{A}=\int_L \frac{\mu_0 Id\mathbf{l}}{4\pi R},\ \mathbf{A}=\int_S \frac{\mu_0\mathbf{K}dS}{4\pi R},\ \mathbf{A}=\int_v \frac{\mu_0\mathbf{J}dv}{4\pi R},\ \Psi=\oint_L \mathbf{A}\cdot d\mathbf{l},\ \mathbf{F}=Q(\mathbf{E}+\mathbf{u}\times\mathbf{B}),\ d\mathbf{F}=Id\mathbf{l}\times\mathbf{B},\ \mathbf{B}_{1n}=\mathbf{B}_{2n},$$

$$(\mathbf{H}_1-\mathbf{H}_2)\times\mathbf{a}_{n12}=\mathbf{K},\ \mathbf{H}_{1t}=\mathbf{H}_{2t},\ \frac{\tan\theta_1}{\tan\theta_2}=\frac{\mu_1}{\mu_2},\ L=\frac{\lambda}{I}=\frac{N\psi}{I},\ M_{12}=\frac{\lambda_{12}}{I_2}=\frac{N_1\psi_{12}}{I_2},\ W_m=\frac{1}{2}\int \mathbf{B}\cdot\mathbf{H}dv=\frac{1}{2}\int \mu H^2 dv$$

WAVES AND APPLICATIONS:

$$V_{emf}=-\frac{d\psi}{dt}\quad ,V_{emf}=\oint_L \mathbf{E}\cdot d\mathbf{l}=-\int_S \frac{\partial\mathbf{B}}{\partial t}\cdot d\mathbf{S}\quad ,\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}\quad ,V_{emf}=\oint_L \mathbf{E}_m\cdot d\mathbf{l}=\oint_L (\mathbf{u}\times\mathbf{B})\cdot d\mathbf{l}$$

$$V_{emf}=\oint_L \mathbf{E}\cdot d\mathbf{l}=-\int_S \frac{\partial\mathbf{B}}{\partial t}\cdot d\mathbf{S}+\oint_L (\mathbf{u}\times\mathbf{B})\cdot d\mathbf{l}\quad ,\mathbf{J}_d=\frac{\partial\mathbf{D}}{dt}\quad ,\nabla\times\mathbf{H}=\mathbf{J}+\frac{\partial\mathbf{D}}{dt}\quad ,\beta=\frac{2\pi}{\lambda},\ \underline{\gamma}=\alpha+j\beta$$

$$\alpha=\omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2}-1\right]},\quad \beta=\omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2}+1\right]},\ \mathbf{E}(z,t)=E_0e^{-\alpha z}\cos(\omega t-\beta z)\mathbf{a}_x$$

$$|\underline{\eta}|=\frac{\sqrt{\mu/\varepsilon}}{\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4}},\quad \tan 2\theta_\eta=\frac{\sigma}{\omega\varepsilon},\ \mathbf{H}=\frac{E_0}{|\underline{\eta}|}e^{-\alpha z}\cos(\omega t-\beta z-\theta_\eta)\mathbf{a}_y,\ \tan\theta=\frac{\sigma}{\omega\varepsilon},\ \mathbf{a}_E\times\mathbf{a}_H=\mathbf{a}_k$$

$$\eta_0=\sqrt{\frac{\mu_0}{\varepsilon_0}}=120\pi\approx 377\Omega,\ p(t)=\mathbf{E}\times\mathbf{H},\ p_{ave}(z)=\frac{1}{2}\mathrm{Re}(\mathbf{E}_s\times\mathbf{H}^*_s),\ p_{ave}(z)=\frac{E_0^2}{2|\underline{\eta}|}e^{-2\alpha z}\cos\theta_\eta\mathbf{a}_z,\ P_{ave}=\int_S p_{ave}\cdot d\mathbf{S},$$

$$\Gamma=\frac{E_{ro}}{E_{io}}=\frac{\eta_2-\eta_1}{\eta_2+\eta_1},\ \tau=\frac{E_{to}}{E_{io}}=\frac{2\eta_2}{\eta_2+\eta_1},\ s=\frac{|\mathbf{E}_1|_{\max}}{|\mathbf{E}_1|_{\min}}=\frac{|\mathbf{H}_1|_{\max}}{|\mathbf{H}_1|_{\min}}=\frac{1+|\Gamma|}{1-|\Gamma|},\quad k_i\sin\theta_i=k_t\sin\theta_t,$$

$$\Gamma_\parallel=\frac{E_{ro}}{E_{io}}=\frac{\eta_2\cos\theta_t-\eta_1\cos\theta_i}{\eta_2\cos\theta_t+\eta_1\cos\theta_i},\ \tau_\parallel=\frac{E_{to}}{E_{io}}=\frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t+\eta_1\cos\theta_i},\ \sin^2\theta_{B\parallel}=\frac{1-\mu_2\varepsilon_1/\mu_1\varepsilon_2}{1-(\varepsilon_1/\varepsilon_2)^2},$$

$$\Gamma_\perp=\frac{E_{ro}}{E_{io}}=\frac{\eta_2\cos\theta_i-\eta_1\cos\theta_t}{\eta_2\cos\theta_i+\eta_1\cos\theta_t},\ \tau_\perp=\frac{E_{to}}{E_{io}}=\frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i+\eta_1\cos\theta_t},\ \sin^2\theta_{B\perp}=\frac{1-\mu_1\varepsilon_2/\mu_2\varepsilon_1}{1-(\mu_1/\mu_2)^2}$$

$\omega = \beta c$

$$S=\frac{|V_{max}|}{|V_{min}|}=\frac{1+|\Gamma|}{1-|\Gamma|}$$

$$\Gamma=\frac{Z_L-Z_0}{Z_L+Z_0}$$

$$y(x,t)=Ae^{-\alpha x}\cos(\omega t-\beta x+\phi_0)$$

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Antenna and Radar formula

Dipole
Solid angle:

$$\Omega_p = \iint_{4\pi} F(\theta,\phi)\, d\Omega$$

Directivity:

$$D = \frac{4\pi}{\Omega_p} \quad \text{or} \quad D = \frac{4\pi A_e}{\lambda^2}$$

Shorted dipole

$$S_0 = \frac{15\pi I_0^2}{R^2}\left(\frac{l}{\lambda}\right)^2$$

$$R_{rad} = 80\pi^2\left(\frac{l}{\lambda}\right)^2.$$

Hertzian monopole

$$R_{rad} = 80\pi^2\left[\frac{dl}{\lambda}\right]^2$$

$$P_{rad} = \frac{1}{2} I_o^2 R_{rad}$$

Half –wave dipole

$$\widetilde{E}_\theta = j\,60 I_0 \left\{\frac{\cos[(\pi/2)\cos\theta]}{\sin\theta}\right\}\left(\frac{e^{-jkR}}{R}\right),$$

$$\widetilde{H}_\phi = \frac{\widetilde{E}_\theta}{\eta_0}.$$

$$|E_{\phi s}| = \frac{\eta_o I_o \cos\left(\frac{\pi}{2}\cos\theta\right)}{2\pi r \sin\theta}$$

$$|H_{\phi s}| = \frac{I_o \cos\left(\frac{\pi}{2}\cos\theta\right)}{2\pi r \sin\theta}$$

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<u>For Transmission line</u>

	Propagation Constant $\gamma = \alpha + j\beta$	Phase Velocity u_p	Characteristic Impedance Z_0
General case	$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$	$u_p = \omega/\beta$	$Z_0 = \sqrt{\dfrac{(R' + j\omega L')}{(G' + j\omega C')}}$
Lossless ($R' = G' = 0$)	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = \sqrt{L'/C'}$
Lossless coaxial	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = (60/\sqrt{\varepsilon_r})\ln(b/a)$
Lossless two-wire	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = (120/\sqrt{\varepsilon_r}) \cdot \ln[(D/d) + \sqrt{(D/d)^2 - 1}]$ $Z_0 \simeq (120/\sqrt{\varepsilon_r})\ln(2D/d)$, if $D \gg d$
Lossless parallel-plate	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = (120\pi/\sqrt{\varepsilon_r})(h/w)$

Notes: (1) $\mu = \mu_0$, $\varepsilon = \varepsilon_r\varepsilon_0$, $c = 1/\sqrt{\mu_0\varepsilon_0}$, and $\sqrt{\mu_0/\varepsilon_0} \simeq (120\pi)\ \Omega$, where ε_r is the relative permittivity of insulating material. (2) For coaxial line, a and b are radii of inner and outer conductors. (3) For two-wire line, d = wire diameter and D = separation between wire centers. (4) For parallel-plate line, w = width of plate and h = separation between the plates.

<u>Distortionless line</u>

$$\gamma = \sqrt{RG} + j\omega\sqrt{LC}$$

$$\frac{R}{L} = \frac{G}{C}, \quad Z_o = \sqrt{\frac{L}{C}}$$

<u>Open-circuited line</u>

$$\widetilde{V}_{oc}(d) = V_0^+[e^{j\beta d} + e^{-j\beta d}] = 2V_0^+ \cos\beta d,$$

$$\widetilde{I}_{oc}(d) = \frac{V_0^+}{Z_0}[e^{j\beta d} - e^{-j\beta d}] = \frac{2jV_0^+}{Z_0}\sin\beta d,$$

$$Z_{in}^{oc} = \frac{\widetilde{V}_{oc}(l)}{\widetilde{I}_{oc}(l)} = -jZ_0 \cot\beta l.$$

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<u>Short-circuited line</u>

$$\widetilde{V}_{sc}(d) = V_0^+[e^{j\beta d} - e^{-j\beta d}] = 2jV_0^+ \sin\beta d,$$

$$\widetilde{I}_{sc}(d) = \frac{V_0^+}{Z_0}[e^{j\beta d} + e^{-j\beta d}] = \frac{2V_0^+}{Z_0}\cos\beta d,$$

$$Z_{sc}(d) = \frac{\widetilde{V}_{sc}(d)}{\widetilde{I}_{sc}(d)} = jZ_0 \tan\beta d.$$

$$j\omega L_{eq} = jZ_0 \tan\beta l, \qquad \text{if } \tan\beta l \geq 0$$

$$\frac{1}{j\omega C_{eq}} = jZ_0 \tan\beta l, \qquad \text{if } \tan\beta l \leq 0$$

$$Z_{in} = Z_o\left[\frac{Z_L + jZ_o \tan\beta\ell}{Z_o + jZ_L \tan\beta\ell}\right]$$

$$Z_{in} = Z_o\left[\frac{Z_L + Z_o \tanh\gamma\ell}{Z_o + Z_L \tanh\gamma\ell}\right]$$

$$V_o = \frac{Z_{in}}{Z_{in} + Z_g} V_g, \qquad I_o = \frac{V_g}{Z_{in} + Z_g}$$

$$V_o = V_L e^{j\beta l}$$

For a bistatic radar (one in which the transmitting and receiving antennas are separated), the power received is given by

$$P_r = \frac{G_{dt}G_{dr}}{4\pi}\left[\frac{\lambda}{4\pi r_1 r_2}\right]^2 \sigma P_{rad}$$

For a monostatic radar, $r_1 = r_2 = r$ and $G_{dt} = G_{dr}$.

$$P_{rec} = P_t G_t G_r \left(\frac{\lambda}{4\pi R}\right)^2$$

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