[CRT01]

UNIVERSITY OF BOLTON

CREATIVE TECHNOLOGIES

BSC (HONS) GAMES PROGRAMMING

SEMESTER TWO EXAMINATION 2021/2022

GAME DYNAMICS

MODULE NO: GAP5006

Date: Tuesday 17th May 2022 **Time: 14:00 – 16:00**

INSTRUCTIONS TO CANDIDATES:

There are SIX questions on this examination.

Answer FOUR questions

Calculators may be used for this examination.

Note: Formula sheets are attached at the rear of the examination.

Where necessary, assume acceleration due to gravity = 9.8 m/s^2.

Question 1

a) Within a game, a thin bullet is fired from the point (6, 3, -5) towards point (8, 2.5, -3). Determine the vector equation of the line passing through these points. [4 marks]

b) Using the equation of the line in a), calculate the shortest distance of the bullet from a point at (7, 2.7, -3.5). [8 marks]

c) Another object's vertex within the game is moving on a trajectory given by the line, $\bar{r} = 2i + j - 0.9k + \lambda(2.2i + 1.2j - 2.5k)$. What is the closest possible distance the vertex of this object could be to the bullet on the equation of the line in a)? [8 marks]

d) In a game, the Enemy fires at the Player if its position is *in front* of the enemy position. Outline how to calculate whether the Player position is in front of the Enemy position, when the Enemy is facing a specified direction, using vectors. [5 marks]

Question 2

a) One face of a game object is represented within the game as a plane. Three points on the object's face are given by: (2, -4, 10), (-1, -3.5, 8) and (2.5, -3, 7). What would the Cartesian equation of the plane be? [8 marks]

b) What is the shortest distance of the game object specified in a) from the origin? [4 marks]

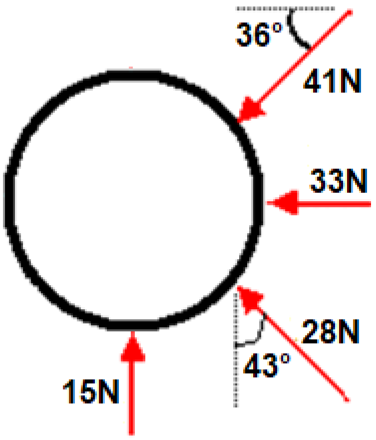
c) The vertex of another game object is at point (6, -2, 7.5). What is the shortest distance between the vertex and the object in a)? [5 marks]

d) What is the 'meaning' of the distance value in c) being positive or negative? [2 marks]

e) During a game, a ray cast is made given by the line, $\bar{r} = 0.1i + 1.1j + 0.2k + \lambda(i - 2j + 3.4k)$. At what point would the ray cast intersect with the object specified in a)? [6 marks]

Question 3

A body has a number of forces acting on it – all acting through the body's centre of mass – as shown below.



a) Calculate the magnitude of the resultant force on the body. [13 marks]

b) Calculate the direction of the resultant force on the body. [4 marks]

c) The body has a mass of 80kg. Assuming no losses, what would be the acceleration of the body, after the forces in a) are applied, and in what direction would it act? [4 marks]

d) If the body was initially at rest, and the resultant force was applied through the centre of mass, and assuming constant acceleration and no losses, how far would the body move in 0.5 seconds? [4 marks]

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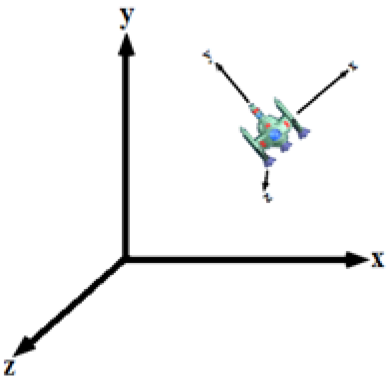
<u>Question 4</u>

An object's vertices are specified in object coordinates and the object is translated and rotated from the world space / coordinates, as shown below.



The object above has a vertex at (1.1, 0.6, -0.5), in object space, and the object was translated by (3, 1.9, 0.6) in world space, and then rotated <u>anti-clockwise</u> 40° about the z-axis.

a) Specify the translation matrix as a 4x4 matrix. [4 marks]

b) Specify the rotation matrix as a 4x4 matrix. [4 marks]

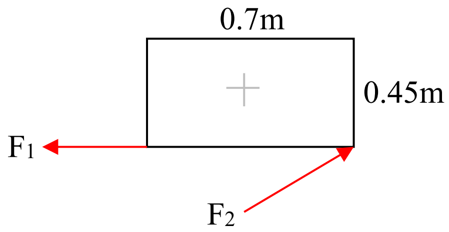
c) Calculate the object's vertex in world space coordinates, using the matrices specified in a) and b). [8 marks]

d) Transformation matrices often include a *homogenous* coordinate. Explain what a *homogenous* coordinate is, and why 4x4 matrices are often used. [4 marks]

e) For rotation, rather than directly calculating a rotation matrix, the rotation is to be specified using quaternions. Specify the above rotation as a *unit* quaternion. [5 marks]

Question 5

A game object is modelled as a cuboid. The cuboid has two forces acting on it, away from its centre of mass in the x and z directions, as shown below.



The cuboid is free to rotate about its centre of mass.

a) Force $\bar{F}_1$ is given by vector *-120i* and Force $\bar{F}_2$ is given by vector *90i-30k*. Assuming that the forces reduce to a single force, calculate the overall torque vector and specify the overall magnitude and rotation axis of the torque. [12 marks]

b) The moment of inertia for a cuboid rotating about its centre of mass, about a principal axis, is given by: $I = \frac{m(l^2+w^2)}{12}$. If the mass of the cube is 800kg, assuming no losses, what would be the angular acceleration of the cuboid? [6 marks]

c) If the cube was initially at rest before the forces were applied, and assuming no losses, how long would it take for the cube to rotate 4°? [7 marks]

Question 6

A car of mass 1.6 tonnes, travelling at 45km/hr, collides into another car, of mass 1.3 tonnes, travelling in the same direction at 29km/hr. The cars separate with a coefficient of restitution of 0.65.

a) What type of collision is this said to be? [1 mark]

b) At what velocity would the cars be travelling after collision? [14 marks]

c) What would be the kinetic energy loss, in Joules, for the collision outlined above? [10 marks]

END OF QUESTIONS

Formula Sheet follows on the next page

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FORMULA SHEET FOR GAME DYNAMICS

Vectors

Dot product: $\bar{a} \cdot \bar{b} = |a||b|\cos\theta$

Cross product: $\bar{a} \times \bar{b} = (|a||b|\sin\theta)\,\hat{n}$ where $\hat{n}$ is a vector at 90° to vectors $\bar{a}$ and $\bar{b}$

Vector equations

Vector equation of a 3D line:	$\bar{r} = \bar{a} + \lambda\bar{b}$
Distance of a point from a 3D line:	$d = \dfrac{\lvert \bar{b} \times (\bar{p} - \bar{a}) \rvert}{\lvert \bar{b} \rvert}$
Shortest distance between 2 skew 3D lines:	$d = \left\lvert \dfrac{(a_1 - a_2) \cdot (b_1 \times b_2)}{\lvert b_1 \times b_2 \rvert} \right\rvert$
Cartesian Equation of a plane:	$ax + by + cz = D$
Distance of a plane from the origin:	$d = \dfrac{D}{\lvert \bar{n} \rvert}$
Distance of a point from a plane	$d = \dfrac{\bar{p} \cdot \bar{n} - D}{\lvert \bar{n} \rvert}$
Intersection of a 3D line on a plane:	$\lambda = \dfrac{D - \bar{a} \cdot \bar{n}}{\bar{b} \cdot \bar{n}}$

Rotation using matrices

For a rotation of θ about the **x-axis**: $R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$

For a rotation of θ about the **y-axis**: $R = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$

For a rotation of θ about the **z-axis**: $R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Quaternions

Unit quaternion, $q = \cos\frac{\theta}{2} + (ai + bj + ck)\sin\frac{\theta}{2}$

Equations of motion

Linear equation of motion	Angular equation of motion
$v_{avg} = s / t$	$\omega_{avg} = \theta / t$
$v = u + at$	$\omega = \omega_0 + \alpha t$
$s = ut + ½at^2$	$\theta = \omega_0 t + ½\alpha t^2$
$v^2 = u^2 + 2as$	$\omega^2 = \omega_0^2 + 2\alpha\theta$

ω_0 = initial angular velocity
$v = \omega r$ and $a = \alpha r$

Angular motion

Torque, $T = Fr$ where F is the resultant force applied and r is the radius at which it acts.

$T = I\alpha$ where I is the moment of Inertia about the rotation axis and α is the angular acceleration

Forces

Resultant force, $F = ma$; where m = mass and a = acceleration

Centripetal force, $F = m\omega^2 r$; where ω = angular velocity and r = radius

Conservation of momentum

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ where m_1/m_2 are the masses of body 1 / 2
u_1 / u_2 are the velocities before impact of bodies 1 / 2
v_1 / v_2 are the velocities after impact of bodies 1 / 2
$v_1 – v_2 = -e(u_1 - u_2)$ where e is the coefficient of restitution

Energy

Kinetic energy, $KE = ½mv^2$ where v = velocity

END OF PAPER