OCD009

UNIVERSITY OF BOLTON

OFF CAMPUS DIVISION

WESTERN INTERNATIONAL COLLEGE FZE

BENG(HONS) MECHANICAL ENGINEERING

TRIMESTER TWO EXAMINATION 2021/2022

FINITE ELEMENT AND DIFFERENCE SOLUTIONS

MODULE NUMBER: AME6016

Date: Tuesday 10th May 2022 **Time: 10:00am – 12:00pm**

INSTRUCTIONS TO CANDIDATES:

There are FIVE questions on the paper.

Answer ANY FOUR questions

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

Electronic calculators may be used provided that data and program storage memory is erased or cleared prior to the examination.

CANDIDATES REQUIRE: **A Formula Sheet (attached)**

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Q1. The rectangular beam shown below in **Figure Q1** is rigidly fixed at end A and simply supported at end B. Point load **P** is applied at 3L from support A along the length of the beam. The beam span AB is 4L in total.

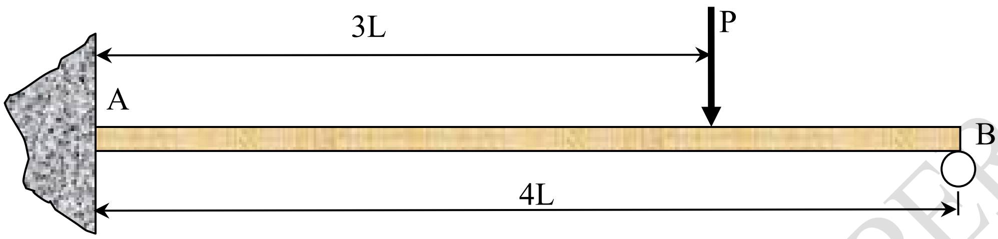

Figure Q1, Beam

Note – B is a simple roller support, whilst end A is built in.
The following data is given:
Young's modulus, E = 200 GNm^{-2}, Moment of inertia, I = 120 x 10^{-9} m^4,
Beam length, L = 1.6 m and Load, P = 10 kN

In answering the questions below, you should split the beam into **four equal sections** and use the Finite Difference method of solution, where:

$$\left(\frac{dy}{dx}\right)_i \approx \frac{1}{2h}\left(y_{i+1} - y_{i-1}\right)$$

$$\left(\frac{d^2y}{dx^2}\right)_i \approx \frac{1}{h^2}\left(y_{i+1} - 2y_i + y_{i-1}\right)$$

$$\left(\frac{d^3y}{dx^3}\right)_i \approx \frac{1}{2h^3}\left(y_{i+2} - 2y_{i+1} + 2y_{i-1} - y_{i-2}\right)$$

$$\left(\frac{d^4y}{dx^4}\right)_i \approx \frac{1}{h^4}\left(y_{i+2} - 4y_{i+1} + 6y_i - 4y_{i-1} + y_{i-2}\right)$$

a) State the Boundary Conditions for the beam. **(2 Marks)**

b) Establish the Bending Moment equations for each node on the beam. **(6 Marks)**

c) Establish the Finite Difference equations for each node. **(7Marks)**

d) Determine the value of the reaction (R_B)at the simple support B **(6 Marks)**

e) Determine the deflection at the mid-point of the beam. **(4 Marks)**

Total 25 marks
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Q2

a) Briefly describe the general steps of the finite element method and list five typical areas of engineering where the finite element method is applied.

(8 Marks)

b) For the spring assemblages shown in **Figure Q2** below, the spring are arranged in series and parallel with node 1, 2 and 4 as fixed and node 3 restricted in moving in x direction only. Use the direct stiffness method for problems.

Determine the following,

1) Using the connectivity table establish the global stiffness matrix for the spring assemblages. **(10 Marks)**

2) Nodal displacements at the junction 3 **(7 Marks)**

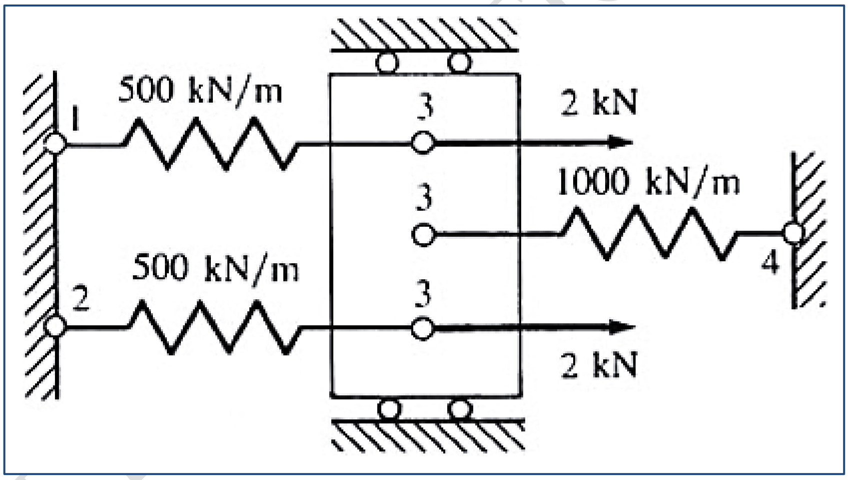

Figure Q2: Spring assemblages

Total 25 marks

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Q3

The aluminum and steel pipes shown in **Figure Q3** below are fastened to rigid supports at the ends, Steel is having twice the area of aluminium.

a) Develop the individual stiffness matrix for Steel and Aluminium **(5 Marks)**

b) Develop the global stiffness matrix using the method of superposition **(5 Marks)**

c) Determine the displacement at nodes **(5 Marks)**

d) Determine the stresses in the aluminum and steel pipes **(5 Marks)**

e) Critically analyse the results obtained. **(5 marks)**

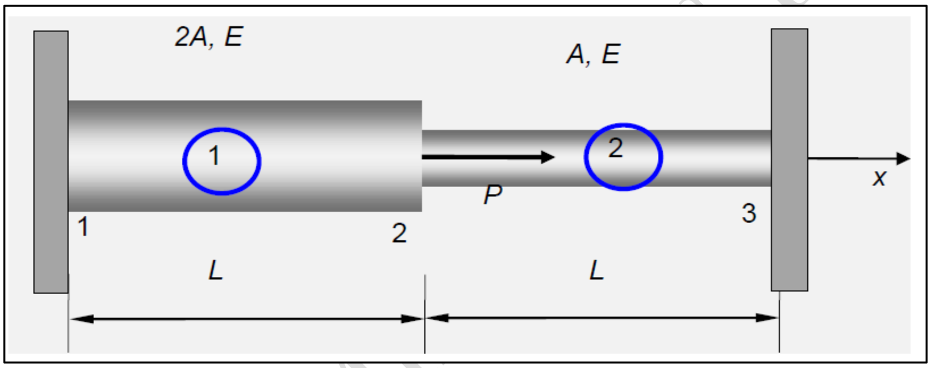

Figure Q3: Assembly of steel and Aluminum pipes.

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Q4

An induction furnace wall is made up of three layers which include inside, middle and outer layer with thermal conductivity K_1= 8.5W/mK, K_2= 0.25W/mK, K_3= 0.088W/mK, convective heat transfer coefficient, h=45W/m^{2}K, outside temperature T_∞=30^{0}C, as shown in **Figure Q4** below, where T_1 is the internal temperature of the furnace, T_2, T_3 are the intermediate temperature of the furnace walls and T_4 is the temperature at the last node in Celsius.

Determine the following.

a) Individual stiffness matrix for the heat conduction. **(9 Marks)**

b) Global stiffness matrix for the system. **(6 Marks)**

c) Nodal Temperatures. **(10 Marks)**

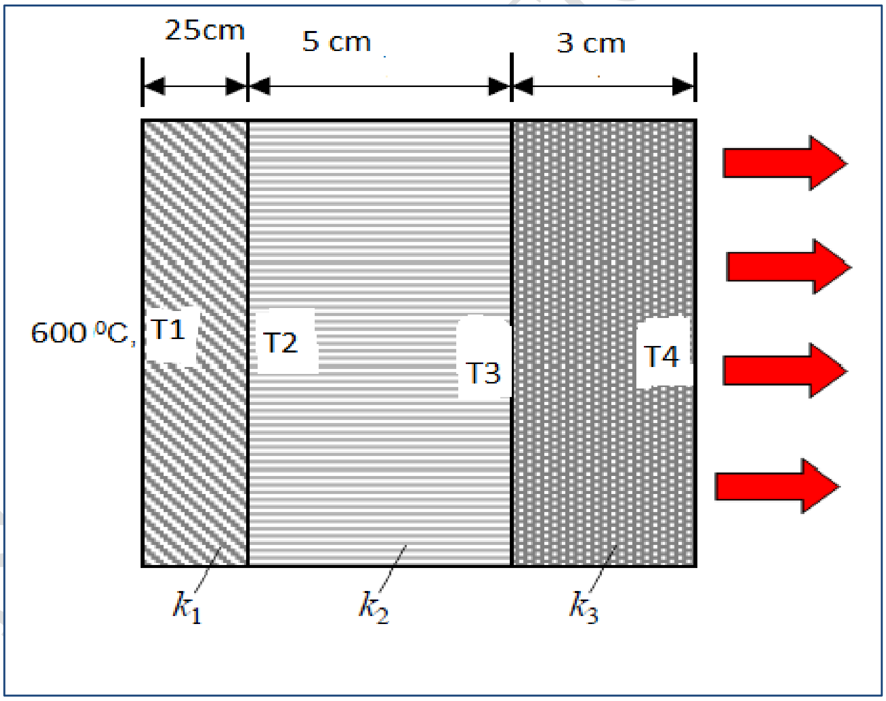

Figure Q4: Induction furnace with three layers.

Total 25 marks
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Q5.

For the bar shown in the Figure Q5 below, with length 2L, modulus of Elasticity, E, mass density ρ, and cross-sectional area A, using the lumped mass matrix determine the following.

a) Discretise the element into two elements. **(5 Marks)**

b) Using direct stiffness matrix develop the global stiffness matrix. **(5 Marks)**

c) Develop the global mass matrix. **(5 Marks)**

d) The first two natural frequencies of the system. **(10 Marks)**

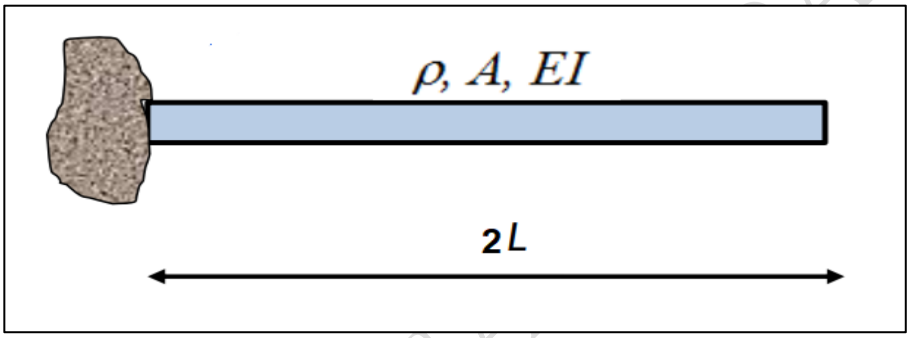

Figure Q5, One-dimensional bar used for natural frequency determination

Total 25 marks

END OF QUESTIONS

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FORMULA SHEET

FINITE ELEMENT AND DIFFERENCE SOLUTIONS

Finite Difference Equations for Beam Deflection:

$$\left(\frac{dy}{dx}\right)_i \approx \frac{1}{2h}\left(y_{i+1} - y_{i-1}\right)$$

$$\left(\frac{d^2y}{dx^2}\right)_i \approx \frac{1}{h^2}\left(y_{i+1} - 2y_i + y_{i-1}\right)$$

$$\left(\frac{d^3y}{dx^3}\right)_i \approx \frac{1}{2h^3}\left(y_{i+2} - 2y_{i+1} + 2y_{i-1} - y_{i-2}\right)$$

$$\left(\frac{d^4y}{dx^4}\right)_i \approx \frac{1}{h^4}\left(y_{i+2} - 4y_{i+1} + 6y_i - 4y_{i-1} + y_{i-2}\right)$$

Stiffness matrix for heat conduction.

$$[\boldsymbol{K_C}] = \frac{\boldsymbol{AK}}{\boldsymbol{l_e}}\begin{bmatrix} \mathbf{1} & \mathbf{-1} \\ \mathbf{-1} & \mathbf{1} \end{bmatrix}$$

Stiffness matrix due to convection.

$$[K_h] = hA\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Force vector due to free end convection

$$[\mathrm{F_h}] = \mathrm{hA\,T_\infty}\begin{Bmatrix} 0 \\ 1 \end{Bmatrix}$$

Element conduction matrix.

$$[k] = \frac{AK_{xx}}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{hPL}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

FORMULA SHEET CONTINUED OVER THE PAGE…..

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Elemental Force Matrix.

$$\{f\} = \frac{QAL + q^* PL + hT_\infty PL}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

Stiffness Matrix:

$$\mathbf{K} = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

Elemental stress.

$$\underline{\sigma} = \underline{C}'\underline{d} \qquad \underline{C}' = \frac{E}{L}[-C \quad -S \quad C \quad S]$$

Lumped Mass Matrix

$$[\hat{m}] = \frac{\rho AL}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Frequency.

$$f_1 = \omega_1 / 2\pi$$

$$|\underline{K} - \omega^2 \underline{M}| = 0$$

END OF FORMULA SHEET

END OF PAPER