

**UNIVERSITY OF BOLTON**  
**SCHOOL OF ENGINEERING**  
**MSC SYSTEM ENGINEERING**  
**SEMESTER ONE EXAMINATION 2019/2020**  
**SIGNAL PROCESSING**  
**MODULE NO: EEM7011**

Date: Wednesday 15<sup>th</sup> January 2020      Time: 14:00 – 16:00

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**INSTRUCTIONS TO CANDIDATES:**

There are **SIX** questions.

Answer **FOUR** questions.

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

This examination paper carries a total of 100 marks.

All working must be shown. A numerical solution to a question obtained by programming an electronic calculator will not be accepted.

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### Question 1

- (a). Contrast the different properties of ; Bessel, Butterworth and Chebyshev filters, giving examples of suitable applications. **(6 marks)**
- (b). Refer to Table Q1 and calculate the components values, for a low-pass 5-order passive Butterworth filter design. The filter should have a 3dB frequency of 500MHz and be designed for use in a  $50 \Omega$  source and load circuit, sketch the design. **(7 marks)**
- (c). Using Table Q1, design a high-pass passive filter circuit. **(7 marks)**
- (d). Describe how low pass filter can be modified to become a band stop filter. **(5 marks)**

**Table Q1.** Butterworth normalised parameters

| k n →<br>↓ | 2      | 3      | 4      | 5      | 6      |
|------------|--------|--------|--------|--------|--------|
| 1          | 1.4142 | 1.0000 | 0.7654 | 0.6180 | 0.5176 |
| 2          | 1.4142 | 2.0000 | 1.8478 | 1.6810 | 1.4142 |
| 3          |        | 1.0000 | 1.8478 | 2.0000 | 1.9319 |
| 4          |        |        | 0.7654 | 1.6810 | 1.9319 |
| 5          |        |        |        | 0.6180 | 1.4142 |
| 6          |        |        |        |        | 0.5176 |

### Question 2

It is required to design a digital filter to replace an analogue filter with the following transfer function;  $H(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$

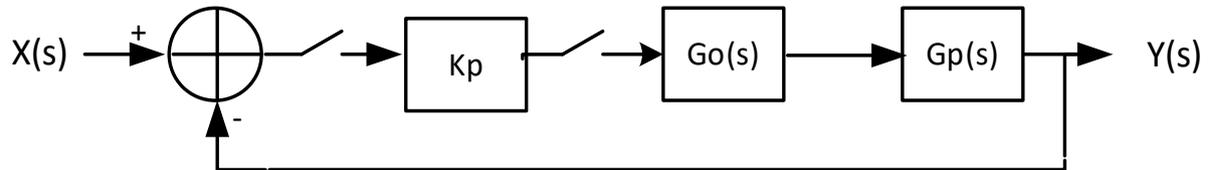
- (a). Using the BZT method derive the digital transfer function  $H(z)$ , assuming a 3dB cut off frequency of 150Hz and a sampling rate of 1.28kHz, if  $s = \frac{z-1}{z+1}$ . **(10 marks)**
- (b). Show your design for  $H(z)$  using delays and feedback. **(10 marks)**
- (c). Derive the recurrence equation for  $y[n]$ . **(5 marks)**

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**Question 3**

A closed loop control system is shown in the block diagram below:



Where the analogue control law  $K_p = 4(2s + 1)/(10s + 1)$  and the plant  $G_p = (0.125)/(s + 0.125)$ .

- (a). It is recommended to replace the analogue controller  $K_p$  with a digital controller with sampling time  $T = 0.2$  seconds, design that digital controller. **(5 marks)**
- (b). Determine the closed loop transfer function in the z domain assuming  $G_o$  is a zero order hold. **(10 marks)**
- (c). Analyse the stability of the closed loop digital system in (b). **(5 marks)**
- (d). Calculate the discrete-time response of the closed system to a unit impulse input For the first four steps. **(5 marks)**

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**Question 4**

(a) Describe the difference between stable and unstable system **[2 Marks]**

(b) In the design of a system to be used in detecting heartrate pattern of unwell patients, a biomedical engineer desires to show a maximum voltage response of 1V when a heartrate pulse is received and discretely zeros otherwise. If the output signal sequence of the patient's heartrate is given by

$$x[n] = 7 \left(\frac{1}{3}\right)^n u[n] - 6 \left(\frac{1}{2}\right)^n u[n]$$

where  $u[n]$  is a unit step function applied to ensure unity voltage when the pulse is received and zero otherwise and  $u[n]$  given by

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

i). Find the z-transform of the signal **[10 Marks]**

ii). Determine the poles **[5 Marks]**

iii). Determine the zeros **[5 Marks]**

iv). Using an ROC plot, determine if the system is stable or unstable **[3 Marks]**

**Question 5**

(a) Explain why analogue filters are usually referred to as IIR filters **[2 Marks]**

(b) Differentiate, with one example each, between causal and non-causal signals. **[4 Marks]**

(c) Identify any four different types of signals processed in frequency domain. **[4 Marks]**

(d) i) State any four methods of designing IIR filter **[4 Marks]**

ii) To approximate the transfer function of an analogue filter to realise its digital IIR filter equivalent, consider the following transfer function

$$H(s) = \frac{2}{1 + s/\omega_c}$$

determine the equivalent z-transform transfer function using bilinear transform method at  $\omega_c = 12 \text{ rad/s}$  and 1.6Hz sampling frequency. **[11 Marks]**

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**Question 6**

(a) State any two difference between recursive and non-recursive filters.

**[2 Marks]**

(b) State any four advantages of frequency domain analysis over time domain analysis.

**[4 Marks]**

(c) The discrete Fourier transform of a signal  $x[n]$  is given by

$$X(k) = \sum_{n=0}^{N-1} x_n e^{-j\frac{2\pi nk}{N}}, \quad \forall k = 0, 1, \dots, N-1$$

where  $k$  and  $n$  are the frequency and time indices, respectively. Considering a signal sequence  $x[n] = \{0, 1, 1, 0\}$ , find the DFT of the signal.

**[8 Marks]**

(d) Both Z-transform and Fourier transform are generally used for signal processing in discrete-time domain, explain why Z-transfer may be preferred than Fourier transform.

**[4 Marks]**

(e) Using Z-transform method, find the frequency response of the follows system

$$y[n] = \frac{1}{6}x[n] + \frac{1}{3}x[n-1] + \frac{1}{6}x[n-2]$$

**[7 Marks]**

**END OF QUESTIONS**

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### Formula Sheet

#### A Table of Basic Laplace and Z transforms

| Time f (t)                 | Laplace F(s)                        | Z transforms   |
|----------------------------|-------------------------------------|--|
| 1. $\delta[t]$             | 1                                   | 1  |
| 2. $u(t)$                  | $\frac{1}{s}$                       | $\frac{z}{z-1}$  |
| 3. $t$                     | $\frac{1}{s^2}$                     | $\frac{Tz}{(z-1)^2}$   |
| 4. $e^{-at}$               | $\frac{1}{s+a}$                     | $\frac{z}{z-e^{-aT}}$  |
| 5. $\frac{1}{1-e^{-at}}$   | $\frac{1}{s+a}$                     | $\frac{z(1-e^{-aT})}{z-1-e^{-aT}}$   |
| 6. $\sin \omega t$         | $\frac{\omega}{s^2 + \omega^2}$     | $\frac{z \sin \omega T}{z^2 - 2z \cos \omega T + 1}$                         |
| 7. $\cos \omega t$         | $\frac{s}{s^2 + \omega^2}$          | $\frac{z^2 - z \cos \omega T}{z^2 - 2z \cos \omega T + 1}$                   |
| 8. $e^{-at} \sin \omega t$ | $\frac{\omega}{(s+a)^2 + \omega^2}$ | $\frac{ze^{-aT} \sin \omega T}{z^2 - 2ze^{-aT} \cos \omega T + e^{-2aT}}$    |
| 9. $e^{-at} \cos \omega t$ | $\frac{s+a}{(s+a)^2 + \omega^2}$    | $\frac{z - e^{-aT} \cos \omega T}{z^2 - 2ze^{-aT} \cos \omega T + e^{-2aT}}$ |
| 10. $\sinh \omega t$       | $\frac{\omega}{s^2 - \omega^2}$     | $\frac{z \sinh \omega T}{z^2 - 2z \cosh \omega T + 1}$                       |
| 11. $\cosh \omega t$       | $\frac{s}{s^2 - \omega^2}$          | $\frac{z - \cosh \omega T}{z^2 - 2z \cosh \omega T + 1}$                     |

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**A Table of Basic Sampled data and Z Transforms**

| signal $x[n]$                 | z Transform $X(z)$   | Region of Convergence |
|-------------------------------|--|-----------------------|
| 1 $\delta[n]$                 | 1  | all $z$               |
| 2 $u[n]$                      | $\frac{z}{z-1}$  | $ z  > 1$             |
| 3 $\beta^n u[n]$              | $\frac{z}{z-\beta}$  | $ z  >  \beta $       |
| 4 $nu[n]$                     | $\frac{z}{(z-1)^2}$  | $ z  > 1$             |
| 5 $\cos(n\Omega)u[n]$         | $\frac{z^2 - z \cos \Omega}{z^2 - 2z \cos \Omega + 1}$                   | $ z  > 1$             |
| 6. $\sin(n\Omega)u[n]$        | $\frac{z \sin \Omega}{z^2 - 2z \cos \Omega + 1}$                         | $ z  > 1$             |
| 7 $\beta^n \cos(n\Omega)u[n]$ | $\frac{z^2 - \beta z \cos \Omega}{z^2 - 2\beta z \cos \Omega + \beta^2}$ | $ z  >  \beta $       |
| 8 $\beta^n \sin(n\Omega)u[n]$ | $\frac{\beta z \sin \Omega}{z^2 - 2\beta z \cos \Omega + \beta^2}$       | $ z  >  \beta $       |

**END OF PAPER**