UNIVERSITY OF BOLTON

WESTERN INTERNATIONAL COLLEGE FZE

BENG (HONS) ELECTRICAL AND ELECTRONIC ENGINEERING

SEMESTER ONE EXAMINATION 2019/2020

ENGINEERING ELECTROMAGNETISM

MODULE NO: EEE6012

Date: Saturday 11th January 2020 **Time: 1:30pm – 4:00pm**

INSTRUCTIONS TO CANDIDATES:

There are FIVE questions.

Answer any FOUR questions.

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

Q1

a) A vector $\vec{F} = 3x\vec{a_x} + 0.5y^2\,\vec{a_y} + 0.25\,x^2\,y^2\vec{a_z}$ is given at point P (3, 4, 2) in the Cartesian co-ordinate system. Express the vector in spherical co-ordinate.

(10 marks)

b) For the object shown in **Figure-1**, all the points are transformed from Cartesian (x, y, z) to cylindrical (ρ, ϕ, z) form as provided below.

O(0, 0, 0) = O(0, 0, 0)

A (0, 0, 8) = A(0, 0, 8)

B(0, 5, 8) = B $(5, \frac{\pi}{2}, 8)$

C(0, 5, 0) = C$(5, \frac{\pi}{2}, 0)$

D(5, 5, 0) = D$(5, \frac{\pi}{3}, 0)$

E(5, 5, 8) = E$(5, \frac{\pi}{3}, 8)$

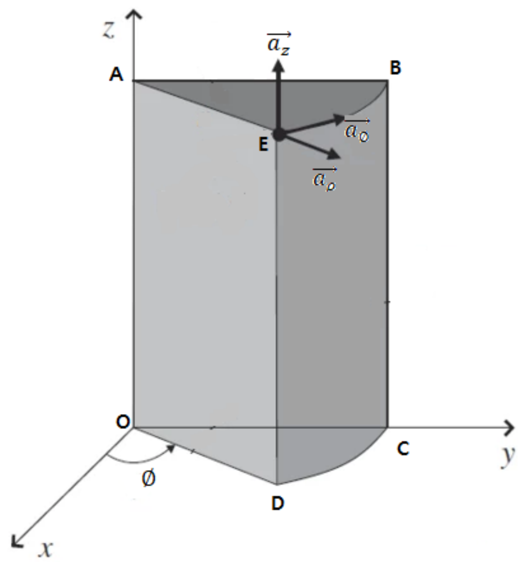

Figure-1

Q1 continued over the page

Q1 continued

Calculate,

(i) The length CD

(3 marks)

(ii) The surface Area BCDE.

(3 marks)

(iii) The volume of the object.

(3 marks)

c) A vector $\vec{T}$ is given by the equation,

$$\vec{T} = \frac{10}{r^2}\cos\theta\,\vec{a_r} + r\sin\theta\,\cos\emptyset\,\vec{a_\theta} + \cos\theta\,\vec{a_\emptyset}$$

Evaluate the Divergence of $\vec{T}$ at $(\frac{1}{2}, \frac{\pi}{4}, 0)$

(6 marks)

Total 25 marks

Q2

a) A DC voltage of 30V applied across the two plates of a parallel plate capacitor having two layers of different dielectric materials as shown in **Figure-2**. The distance between the two plates of the capacitor is 3mm, Area of each plate is 1 m^2, the relative permittivity of the first dielectric material is 5 and the second dielectric material is 1. Determine the voltages across each dielectric in the capacitor

(8 marks)

Q2 continued over the page

Q2 continued

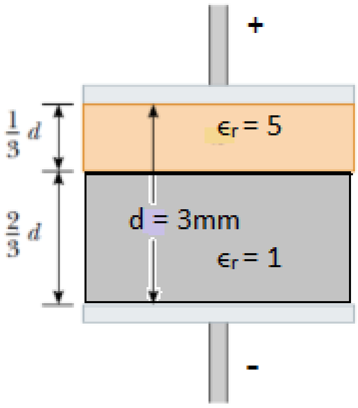

Figure-2

b) A finite sheet of the capacitor has a surface charge density $\rho_s = xy(x^2 + y^2)$ nC/m^2. If the finite sheet has a dimension, $0 \leq x \leq 1, 0 \leq y \leq 1$ and Z=0, Calculate the total charge present in the capacitor.

(8 marks)

c) Two point charges 2mC and 3mC are located at (-2, 3, 1) and (5, 2, -3) in the capacitor. Calculate

(i) The electric force on 7nC located at (0, 2, 1)

(7 marks)

(ii) Electric field intensity at that point

(2 marks)

Total 25 marks

Q3

a) A circular conducting loop of radius 40cm.lies in the xy plane and has a resistance of 20 ohms. If the magnetic flux density in the region is given as

$\vec{B} = 0.2\cos 500t\,\vec{a_x} + 0.75\sin 400t\,\vec{a_y} + 1.2\cos 314t\,\vec{a_z}$ Tesla.

Determine the effective value of the induced current in the loop.

(10 marks)

b) A conducting bar slides feely over two conducting rails as shown in **Figure-3**. Calculate the induced voltage in the bar.

i. If the Bar is stationed at y = 8cm and $\vec{B} = 4\cos 10^6\,t\,\vec{a_z}$ mWb /m^2

(7 marks)

ii. If the Bar slides at a velocity of u = 20 $\vec{a_y}\,m/S$ and $\vec{B} = 4\vec{a_z}\ mWb$/m^2

(4 marks)

iii. Calculate the time varying magnetic flux ϕ for the time varying magnetic field $\vec{B} = 4\cos 10^6\,t\,\vec{a_z}\ mWb$ /m^2

(4 marks)

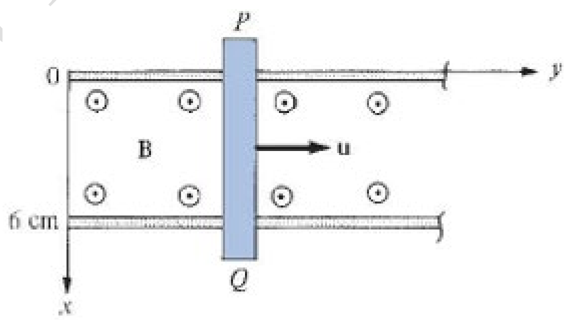

Figure-3

Total 25 marks

Please turn the page

Q4

a) A telephone line has R = 30 Ω/km, L = 100 mH/km, G = 0, and C = 20 μF/km. At f = 1 kHz, obtain:

(i) The characteristic impedance of the line (3 marks)

(ii) The propagation constant (3 marks)

(iii) The phase velocity (4 marks)

b) A certain transmission line 2 m long operating at $\omega = 10^6$ rad/s has $\alpha = 8$ dB/m, $\beta = 1$ rad/m, and $Z_0 = 60 + j40\Omega$. If the line is connected to a source of $10 \angle 0°$ V, $Z_g = 40\ \Omega$ and terminated by a load of $20 + j50\Omega$, determine

(i) The input impedance (4 marks)

(ii) The sending-end current (4 marks)

(iii) The current at the middle of the line (7 marks)

Total 25 marks

Q5

An antenna with an impedance of 40 + j30Ω, is to be matched to a 100 Ω, lossless line with a shorted stub. Determine using smith plot,

(i) The required stub admittance (7 marks)

(ii) The distance between the stub and the antenna (8 marks)

(iii) The stub length (5 marks)

(iv) The standing wave ratio on each segment of the system (5 marks)

Total 25 marks

END OF QUESTIONS

Formula sheet

Coordinate systems:

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \tan^{-1}\frac{\sqrt{x^2 + y^2}}{z}$$

$$\phi = \tan^{-1}\frac{y}{x}$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

Differential Length, Surface and volume:

$$d\mathbf{l} = d\rho\,\mathbf{a}_\rho + \rho\,d\phi\,\mathbf{a}_\phi + dz\,\mathbf{a}_z$$

$$d\mathbf{S} = \rho\,d\phi\,dz\,\mathbf{a}_\rho + d\rho\,dz\,\mathbf{a}_\phi + \rho\,d\rho\,d\phi\,\mathbf{a}_z$$

$$dv = \rho\,d\rho\,d\phi\,dz$$

Vector calculus:

$$\nabla\cdot\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

Formula Sheet continued over the page

Formula Sheet continued

<u>Electrostatics:</u>

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

$$C_T = \frac{C_1 C_2}{C_1 + C_2}$$

$$Q = CV$$

$$D = \frac{Q}{A}$$

$$E = \frac{D}{\epsilon_0 \epsilon_r}$$

V = E × d

$$Q = \int_S \rho_S \, dS$$

$$\varepsilon_o = 8.854 \times 10^{-12} \simeq \frac{10^{-9}}{36\pi} \text{ F/m}$$

$$\vec{\mathbf{F}} = \frac{Q}{4\pi\varepsilon_o} \sum_{k=1}^{N} \frac{Q_k(\mathbf{r} - \mathbf{r}_k)}{|\mathbf{r} - \mathbf{r}_k|^3}$$

$$\vec{E} = \frac{\vec{F}}{q}$$

Formula Sheet continued over the page

Formula Sheet continued

Magnetostatics:

$$d\emptyset = \vec{B}.\overrightarrow{dS}$$

$$\emptyset = \int \vec{B}.\overrightarrow{dS}$$

$$E = 4.44\, Nf\emptyset_m$$

$$V_{\text{emf}} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}$$

$$V_{\text{emf}} = \int_L (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}$$

MAXWELL'S EQUATIONS:

$\nabla .E_s = 0$

$\nabla .H_s = 0$

$\nabla \times H_s = j\omega\varepsilon_0 E_s$

$\nabla \times E_s = -j\omega\mu_0 H_s$

$$\nabla \times \mathbf{A} = \left[\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right]\mathbf{a}_\rho + \left[\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right]\mathbf{a}_\phi + \frac{1}{\rho}\left[\frac{\partial(\rho A_\phi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \phi}\right]\mathbf{a}_z$$

$\omega/\beta = C/\sqrt{(\mu_r\varepsilon_r)}$

$\beta = 2\pi/\lambda$

$E_0/H_0 = \sqrt{(\mu_0\mu_r/\varepsilon_0\varepsilon_r)}$

Formula Sheet continued over the page

Formula Sheet continued

Wave propagation and Transmission lines

$\varepsilon_r = \beta^2 / (\omega^2 \mu_0 \mu_r \varepsilon_0)$
$\eta = \sqrt{(\mu/\varepsilon)}$
$P_{avg} = E_0^2 / 2\eta .\ a_n$
$P_{total} = \int P_{avg} .dS = P_{avg}.\ S.\ a_n$
1 Np = 8.686 dB., J1 radian = J 57.3^0 degrees
Propagation Constant, $\gamma = \alpha + j\beta$

$$\tanh(x \pm jy) = \frac{\sinh 2x}{\cosh 2x + \cos 2y} \pm j \frac{\sin 2y}{\cosh 2x + \cos 2y}$$

$$Z_{in} = Z_o \left(\frac{Z_L + Z_o \tanh \gamma \ell}{Z_o + Z_L \tanh \gamma \ell} \right)$$

$$I(z = 0) = \frac{V_g}{Z_{in} + Z_g}$$

$$V_o = Z_{in} I_o$$

$$V_o^+ = \frac{1}{2}(V_o + Z_o I_o)$$

$$V_o^- = \frac{1}{2}(V_o - Z_o I_o)$$

$$I_s(z = \ell/2) = \frac{V_o^+}{Z_o} e^{-\gamma z} - \frac{V_o^-}{Z_o} e^{\gamma z}$$

$$\text{phase velocity, } v = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu \varepsilon}}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$$

$$\nabla \times \mathbf{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) \hat{\mathbf{x}} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) \hat{\mathbf{y}} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \hat{\mathbf{z}}$$

Formula Sheet continued over the page

Formula sheet continued

Waveguides and Optical Fibres:

$$f_{c_{mn}} = \frac{u'}{2}\sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}}$$

$$u' = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\beta = \omega\sqrt{\mu\varepsilon}\sqrt{1 - \left[\frac{f_c}{f}\right]^2}$$

$$\gamma = j\beta$$

$$\eta_{TM_{mn}} = \eta'\sqrt{1 - \left[\frac{f_c}{f}\right]^2}$$

For the TE_{10} mode

$$\alpha_d = \frac{\sigma\eta'}{2\sqrt{1 - \left[\frac{f_c}{f}\right]^2}}$$

$$f_c = \frac{u'}{2a}$$

$$\eta' \simeq \sqrt{\frac{\mu}{\varepsilon}}$$

$$R_s = \frac{1}{\sigma_c\delta} = \sqrt{\frac{\pi f\mu}{\sigma_c}}$$

For the TE_{10} mode

$$\alpha_c = \frac{2R_s}{b\eta'\sqrt{1 - \left[\frac{f_c}{f}\right]^2}}\left(0.5 + \frac{b}{a}\left[\frac{f_c}{f}\right]^2\right)$$

$$\alpha = \alpha_d + \alpha_c$$

$$P_a = (P_d + P_a)\, e^{-2\alpha z}$$

Numerical aperture, NA = Sin θ_a = $\sqrt{(n_1^2 - n_2^2)}$*

$$V = \pi d\sqrt{(n_1^2 - n_2^2)} \backslash \lambda$$

No: of modes, N = $V^2/2$

$\alpha\ell = 10 \log_{10}[P(0)/P(\ell)]$

END OF PAPER