

**UNIVERSITY OF BOLTON**  
**WESTERN INTERNATIONAL COLLEGE FZE**  
**BENG (HONS) MECHANICAL ENGINEERING**  
**SEMESTER ONE EXAMINATION 2019/2020**  
**ADVANCED MATERIALS & STRUCTURES**  
**MODULE NO AME6012**

Date: Saturday 11<sup>th</sup> January 2020

Time: 10:00 AM – 1:00 PM

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**INSTRUCTIONS TO CANDIDATES:**

There are FIVE questions on this paper.

Answer any FOUR questions only.

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

Electronic calculators may be used provided that data and program storage memory is cleaned prior to the examination.

**CANDIDATES REQUIRE:**

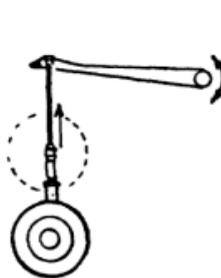
Formula Sheet (attached)

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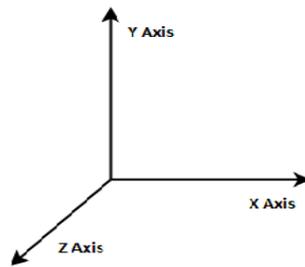
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### Question 1

Part of a landing gear of an aircraft is subjected to the following direct stresses in the x, y and z directions as shown in the **Figure1a**. The stress tensor matrix for the given scenario is shown in **Figure1b**.



**Figure1a.** Aircraft landing gear



$$\sigma = \begin{bmatrix} 30 & 0 & 10 \\ 0 & 0 & 20 \\ 10 & 20 & 0 \end{bmatrix}$$

**Figure1b.** Stress Tensor Matrix

Determine the following:

- Draw the elemental cube showing the stresses acting on it. (5 marks)
- Using this information given above prove that one of the principal stress is a compressive stress of 21 MPa (10 marks)
- the angles relative to xyz co-ordinates and make a sketch showing the direction of these stresses. (5 marks)
- If the yield stress for the material is 320 MPa determine the factor of safety assuming the material follows the Von Misses criterion. (5 marks)

**Total 25 marks**

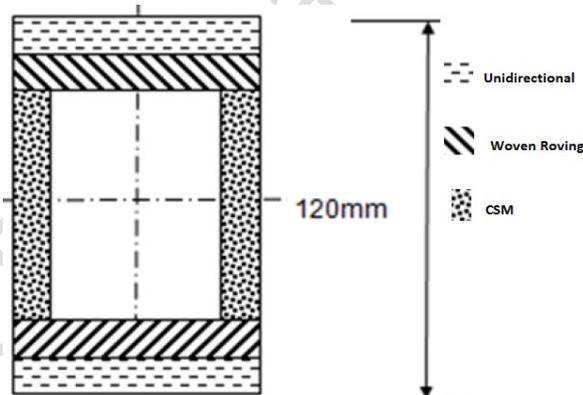
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### Question 2

A 90 X 120mm wide pultruded section fabricated from glass reinforced polyester is shown in **Figure Q2** below with the material specification. The section is used as a cantilever 3 m in length. Beams are to be used to support a mass of 500 Kg and are placed at the midpoint of the cantilever. If the beam is designed to have a maximum design strain of 0.2 %, determine the following using the data from the **Table1**.

- a) Sketch the stress distribution through the depth of each beam and indicate the salient values (15 Marks)
- b) Determine the number of beams used to support the section. (5 Marks)
- c) Sketch the strain diagram for the pultruded section (5 Marks)



**Figure Q2.** Pultruded Cross section of the Beam

**Table 1:** Details of the composite structure

Material	Efficiency Factor, %	Modulus (GPa)	Volume Fraction, %	Thickness, mm
Unidirectional	0.9	65	60	10
Woven Roving	0.5	65	40	10
CSM	0.25	65	30	40
Polyester Resin	-	3		

**Total 25 marks**  
**Please turn the page**

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### **Question 3**

- a) The values of the endurance limits at various stress amplitude levels for low-alloy constructional steel fatigue specimens are given in the **Table 2** below where  $\sigma$  indicates stress value and  $N_f$  for number of cycles for fatigue.

**Table 2 - Stress & number of cycles**

$\sigma$ (MN/m <sup>2</sup> )	$N_f$ (Cycles)
550	1500
510	10050
480	20800
450	50 500
410	125000
380	275000

A similar specimen is subjected to the following programme of cycles at the stress amplitudes stated;  $N_f=3000$  at  $\sigma=510$  MN/m<sup>2</sup>,  $N_f=12000$  at  $\sigma=450$  MN/m<sup>2</sup> and  $N_f=80000$  at  $\sigma=380$  MN/m<sup>2</sup>, after which the sample remained unbroken. Determine the additional cycles the specimen need to withstand at  $\sigma=480$  MN/m<sup>2</sup> prior to failure?

Assume zero mean stress conditions.

(12 marks)

- b) The fatigue behaviour of mild steel specimen under an alternating stress conditions with zero mean stress is given by the expression:

$$\sigma_r^a \cdot N_f = K$$

Where  $\sigma_r$ , is the range of cyclic stress,

$N_f$  is the number of cycles to failure and  $K$  and 'a' are material constants of mild steel.

If it is given that  $N_f = 10^6$  when  $a = 300$  MN/m<sup>2</sup> and  $N_f = 10^8$  when  $a = 200$  MN/m<sup>2</sup>.

Determine the constants  $K$  and 'a' and also find the life of the specimen when subjected to a stress range of 100 MN/m<sup>2</sup>.

(13 marks)

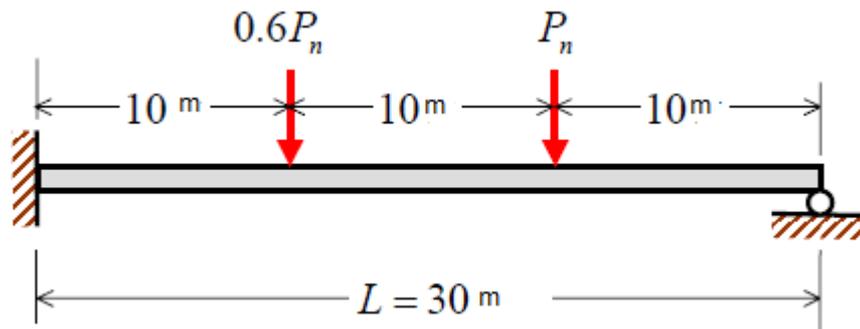
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#### **Question 4**

For the statically indeterminate beam as shown in the **Figure Q4** below, the beam is supported at one end by a fixed support and roller support at the other end. Two point loads at a distance of 10 m from each end is applied for a span of 30 m ( $L$ ). Using the virtual work method answer to the following questions

- Illustrate all the possible collapse mechanism for the given beam considering the forces applied on the member. (5 Marks)
- Determine the plastic limit  $P_n$  (nominal load) considering the external and internal work done in all the possible cases of collapse. (15 Marks)
- Comment on the findings with the reasons for the selection of the collapsed load. (5 Marks)



**Figure Q4** - Indeterminate beam for collapse mechanism

**Total 25 marks**

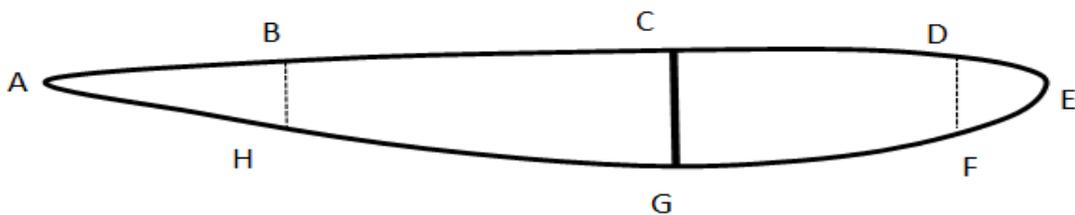
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### **Question 5**

An aluminium aerofoil section of a racing car is shown in **Figure Q5** with a span length of 1 m. Under the worst case scenario the section is subjected to a torque of 770 Nm, answer to the following questions:

- Using the geometric information provided in **Table 2**, evaluate the maximum shear stress and state where this occurs. Assume modulus of rigidity,  $G$  for this material as 27GPa. (13 marks)
- Determine for this condition the angle of twist over 1m span. (3 marks)
- If during a race, the aluminium skin splits at position 'H', analyse the new maximum stress and angle of twist. (7 marks)
- Describe briefly why the value of the angle of twist is an overestimate compared to an actual section. (2 marks)

**Total 25 marks**



**FigureQ5** - Aluminium Aerofoil Section

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**Q5 continued...**

**Table 2 - Geometric Data of Aluminium Aerofoil Section**

Position	Length (mm)	Thickness (mm)
AB	78	1.06
BC	86	1.06
CD	60	1.06
DE	38	1.06
EF	38	1.06
FG	64	1.06
GH	92	1.06
HA	80	1.06
CG	36	2.25

Area	Size (mm <sup>2</sup> )
ABH	960
BCGH	2700
CDFG	2300
DEF	160

**END OF QUESTIONS**

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### Formula Sheet

#### Elasticity – finding the direction vectors

$$\begin{bmatrix} S_x \\ S_y \\ S_z \end{bmatrix} = \begin{pmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{pmatrix} \begin{pmatrix} l \\ m \\ n \end{pmatrix}$$

$$k = \frac{1}{\sqrt{a^2 + b^2 + c^2}}$$

Where a, b and c are the co-factors of the eigenvalue stress tensor.

$$\begin{aligned} l &= ak & l &= \cos\alpha, \\ m &= bk & m &= \cos\theta, \\ n &= ck & n &= \cos\varphi. \end{aligned}$$

### Principal stresses and Mohr's Circle

#### Yield Criterion

$$\tau_{12} = \frac{\sigma_1 - \sigma_2}{2}$$

#### Von Mises

$$\tau_{13} = \frac{\sigma_1 - \sigma_3}{2}$$

#### Tresca

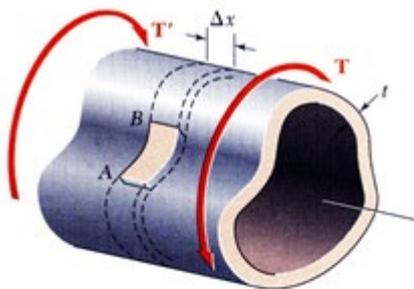
$$\sigma_{von\ Mises} = \frac{1}{\sqrt{2}} \left[ (\sigma_1 - \sigma_2)^2 + \frac{\sigma_2 - \sigma_3}{2} \cdot \frac{\sigma_2 - \sigma_3}{2} + (\sigma_3 - \sigma_1)^2 \right]^{1/2}$$

$$\sigma_{tresca} = 2 \cdot \tau_{max}$$

$$\tau_{max} = \max \left( \frac{|\sigma_1 - \sigma_2|}{2}, \frac{|\sigma_1 - \sigma_3|}{2}, \frac{|\sigma_3 - \sigma_2|}{2} \right)$$

Formula sheet continued...

Torsion in close thin wall cross section (CTW)



- Shear stress varies inversely with thickness

$$\tau = \frac{T}{2tA}$$

- Shear flow q

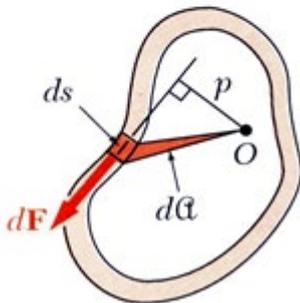
$$q = \tau t$$

- Applied torque T

$$T = 2qA$$

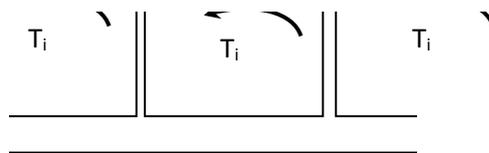
- Angle of twist  $\phi$

$$\phi = \frac{TL}{4A^2G} \oint \frac{ds}{t}$$



Torsion in multi-cells thin wall cross section

- Section considered as an assembly of N tubular sub-sections (compartments), each subjected to torque  $T_i$  as shown in the figure below:



- Total torque

$$T = \sum_{i=1}^n T_i = 2 \sum_{i=1}^n q_i A_i$$

- Common angle of twist for all compartments:

$$\theta = \frac{L}{4GA_i} \oint \frac{q_i - q'}{t(s)} ds$$

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$$\varphi_1 = \frac{L}{2GA_1} \left( \frac{q_1 \ell_1}{t_1} + \frac{(q_1 - q_2) \ell_3}{t_3} \right)$$

$$\varphi_2 = \frac{L}{2GA_2} \left( \frac{q_2 \ell_2}{t_2} + \frac{(q_2 - q_1) \ell_3}{t_3} \right)$$

Where  $q$  is the shear flow of the main compartment,  $q_i$  is the shear flow due to torque in adjacent compartments,  $A_i$  the area of cross-section  $i$ ,  $t$  is the thickness of the cross-section and  $s$  is the circumference of the compartment.

Torsion in open thin wall cross section (OTW)

If  $\frac{b}{t} \geq 10$  then  $\alpha = \beta = \frac{1}{3}$   
 and  $J_\alpha = J_\beta = J = \sum_{i=1}^n \frac{1}{3} b_i t_i^3$

*Shear stress*

$$\tau_{\max} = \frac{T t_{\max}}{J}$$

*Twist angle*

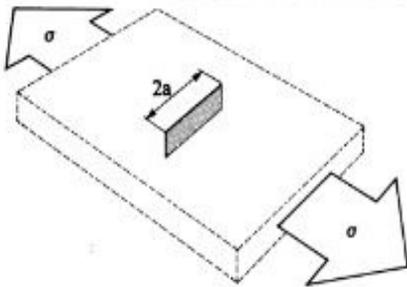
$$\varphi = \frac{LT}{GJ}$$

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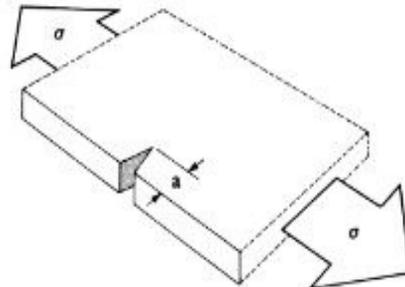
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## Fracture mechanics

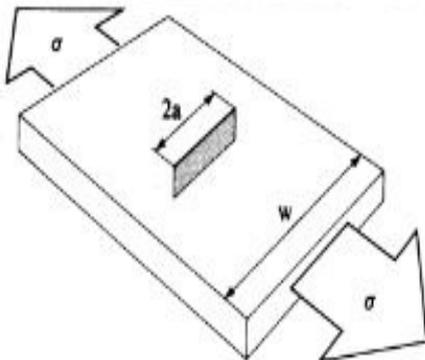
Table: Y values for plates loaded in tension



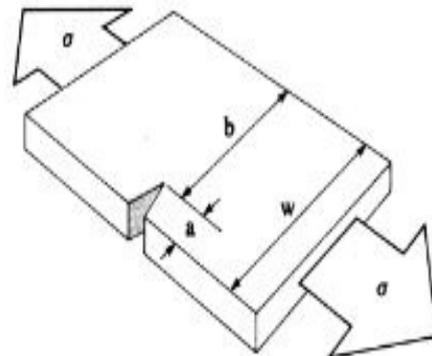
- (1) Through crack of length  $2a$  in an *infinite* plate  
 $Y = 1$



- (2) Edge crack of length  $a$  in an *infinite* plate  
 $Y = 1.12$   
 Because plane strain and plane stress have identical stress fields, this calibration is also for an edge scratch of depth  $a$  on a large body carrying tensile stress  $\sigma$ .



- (3) Through crack of length  $2a$  in a plate of width  $w$ .  
 $Y = \left( \sec \frac{\pi a}{w} \right)^{1/2}, \frac{2a}{w} \leq 0.7$



- (4) Edge crack of length  $a$  in a plate of width  $w$ .  
 $Y = 0.265 \left( \frac{b}{w} \right)^4 + \frac{0.875 + 0.265a/w}{(b/w)^{3/2}}$

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## Life Calculations

$$K = Y\sigma\sqrt{\pi a}$$

$$\frac{da}{dN} = C(\Delta K)^m$$

$$N = \frac{1}{CY^m\sigma_a^m\pi^{\frac{m}{2}}} \left[ \frac{a^{1-\frac{m}{2}}}{1-\frac{m}{2}} \right]_{a_0}^{a_1}$$

## Composite materials

$$E_c = \eta E_f V_f + E_m (1 - V_f)$$

### Miners Rule.

Miners Rule  $\sum \frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} + \dots = 1$

END OF PAPER