[ESS10]

UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING SCIENCES

BEng (HONS) MECHANICAL, ELECTRICAL & ELECTRONIC ENGINEERING

SEMESTER ONE EXAMINATIONS 2019/20

ENGINEERING MODELLING AND ANALYSIS

MODULE NO: AME5014

Date: Wednesday 15th January 2020 **Time: 2:00pm – 4:00pm**

INSTRUCTIONS TO CANDIDATES:

There are EIGHT questions.

Answer ANY FIVE questions only.

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

Electronic calculators may be used provided that data and program storage memory is cleared prior to the examination.

CANDIDATES REQUIRE: **Formula Sheet (attached).**

Q1

The ordinary differential equation (ODE) describing the displacement x(t) in mm in function of time t of a voice box simulator can be modelled approximately by the equation below:

$$\ddot{y}(t) - 15\,\dot{y}(t) + 56\,y(t) = 24$$

Given: $\ddot{y}(t),\ \dot{y}(t)\ and\ y(t)\ all\ equal\ to$ 0 at t = 0,

a) Use the method of Laplace transforms to derive an expression for y(t)

(14 marks)

b) Sketch how y(t) varies with time for the first 5 seconds.

(6 marks)

Total 20 marks

Q2

It can be shown that a simple two degree of freedom electronic device in an electromagnetic field can be described by $\hat{T} = K\vec{\emptyset}$ where: $\hat{T}$ and $\vec{\emptyset}$ are torque and rotation column vectors respectively and K is the stiffness matrix. Using,

$\vec{T} = \begin{pmatrix} 60 \\ -25 \end{pmatrix}$ Nm and $K = \begin{bmatrix} 1700 & -600 \\ -600 & 1900 \end{bmatrix}$Nmm / degree

Calculate the displacement vector $\vec{\emptyset}$ in radian.

Total 20 marks

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Q3

The output speed of a motor ω, in rad/s, is related to the step input angle ϴ, in radian, of the sensor by the following transfer function:

$$\frac{\omega(s)}{\theta(s)} = \frac{15}{4s+2}$$

a) Determine the DC gain K and the time constant τ of the system.

(6 marks)

b) Calculate the speed indicated if the angle of the input sensor is 1.5 radians?

(7 marks)

c) Determine the angle of the input sensor if the speed of the motor reaches 75% of its maximum value.

(7 marks)

Total 20 marks

Q4

a) If $z = x\ sin(y),\ ,$ using the partial differentiation give the value of

$$\frac{\partial^2 z}{\partial y^2} + xy \cdot \frac{\partial^2 z}{\partial x^2}$$

$If\ \ x = \pi\ \ and\ \ y = \pi/4$

(10 marks)

b) Calculate the quantity of a crude oil extracted by a mechanical pump in three dimensions (xyz) that can be expressed by the volume V bounded above by the shape $z=x^2y^2$ and below by the rectangle R = {(x, y): 0 ≤ x ≤ 2, 0 ≤ y ≤ 3}.

(10 marks)

Total 20 marks

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Q5

The stress σ, in MPa, at a point in a body can be described by the following matrix A relative to the global co-ordinate system xyz.

$$A = \begin{bmatrix} 3 & 2 & 0 \\ 4 & 1 & 0 \\ 5 & 0 & 1 \end{bmatrix} \text{MPa}$$

a) Using an appropriate technique, show that the principal Eigen values (principal stresses, Maximum Stresses) at this point are: λ_1 = 5 MPa, λ_2 = 1MPa and λ_3 = -1MPa.

(10 marks)

b) Determine also the associated Eigen vector and the cosine direction of the largest principal stress.

(10 marks)

Total 20 marks

Q6

Part of a valve regular operates at a frequency ω of 1.2 rad/s. If the equation of motion is given by:

$$\ddot{y} + 2\zeta\omega_n\dot{y} + \omega_n^2 y = 0$$

Given: $y \ in \ mm, \quad \zeta = 0.15, \ \omega_n = 1.5\frac{rad}{s}, y(0) = 1mm, \dot{y}(o) = 0\frac{mm}{s}$.

a) Find the expression of the motion of the valve in function of time. **(14 marks)**

b) Sketch how y(t) varies with time for the first 7 seconds. **(6 marks)**

Total 20 marks

Q7

The pressure in a valve varies in relation to angular movement. The table and graph below show this variation. The work done **W** by the system is calculated as follows:

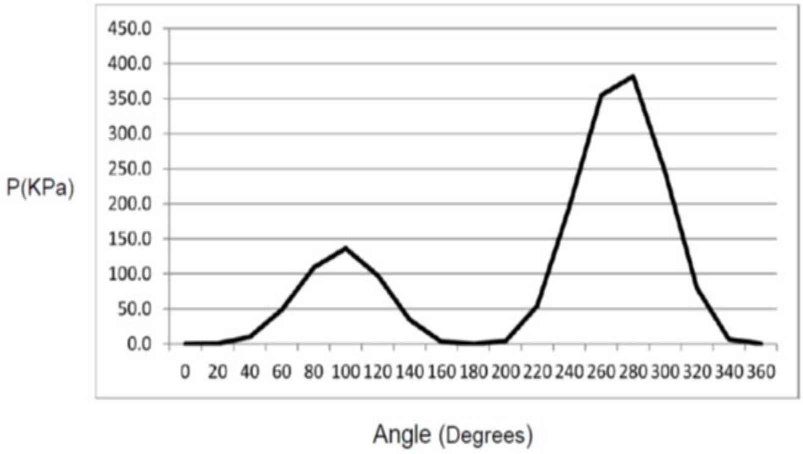
$$\boldsymbol{W = \delta \cdot A}$$

$\boldsymbol{A = \int_{\varphi 1}^{\varphi 2} P\, d\varphi}$, the integral under the curve where $\boldsymbol{P}$ is the pressure in KPa, $\boldsymbol{\varphi}$ is the angle in radians and $\boldsymbol{\delta}$ is the constant in mm^3. If $\boldsymbol{\delta}$ is 2 $\times 10^6$ mm^3, calculate:

a) the work done in one cycle. **(14 Marks)**

b) Also, if it takes one minute for a cycle what is the power rating of the valve? **(6 Marks)**

Angle(Deg)	Pressure(KPa)
0	0.0
20	0.4
40	9.9
60	48.9
80	109.1
100	136.4
120	97.9
140	34.7
160	3.2
180	0.0
200	4.0
220	54.4
240	195.7
260	354.6
280	381.9
300	244.8
320	79.3
340	6.8
360	0.0



Total (20 marks)

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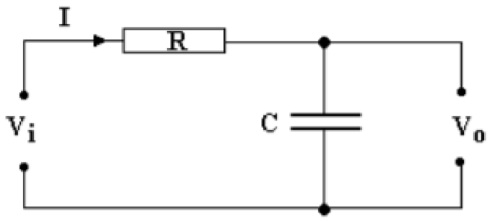
Q8

The following RC circuit is shown in which R= 200Ω and C=15μF.



The voltage V_i =10 Volts. The charge of the capacitor in the circuit is described by the following 1st order differential equation:

$$\frac{dV_o}{dt} = k(V_i - V_o)$$

Given: the coefficient k=1/RC and V_o (0)=0.

a) Calculate the time required if the voltage at the generator is V_o =5 volts.

(7 marks)

b) Calculate the value of V_o after t=0.025s

(4 Marks)

c) Determine the time required for V_o to increase from 3 Volts to 8 volts.

(9 marks)

Total 20 marks

END OF QUESTIONS

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Formula sheet

Partial Fractions

$$\frac{F(x)}{(x+a)(x+b)} = \frac{A}{(x+a)} + \frac{B}{(x+b)}$$

$$\frac{F(x)}{(x+a)(x+b)^2} = \frac{A}{(x+a)} + \frac{B}{(x+b)} + \frac{C}{(x+b)^2}$$

$$\frac{F(x)}{(x^2+a)} = \frac{Ax+B}{(x^2+a)}$$

Small Changes

z = f(u, v, w)

$$\delta z \simeq \frac{\partial z}{\partial u}.\delta u + \frac{\partial z}{\partial v}.\delta v + \frac{\partial z}{\partial w}.\delta w$$

Total Differential

z = f(u, v, w)

$$dz = \frac{\partial z}{\partial u} du + \frac{\partial z}{\partial v} dv + \frac{\partial z}{\partial w} dw$$

Rate of Change

z = f(u, v, w)

$$\frac{dz}{dt} = \frac{\partial z}{\partial u}.\frac{du}{dt} + \frac{\partial z}{\partial v}.\frac{dv}{dt} + \frac{\partial z}{\partial w}.\frac{dw}{dt}$$

Eigenvalues

$$|A - \lambda \mathrm{I}| = 0$$

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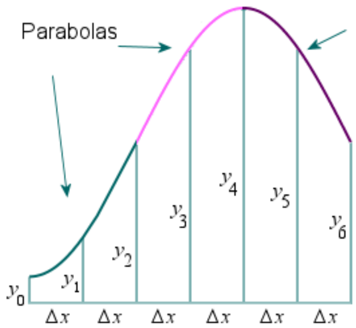
Eigenvectors

$$(A-\lambda_r \mathrm{I})x_r = 0$$

Integration

$$\int u.\frac{dv}{dx}dx = uv - \int v.\frac{du}{dx}dx$$

Simpson's rule
To calculate the area under the curve which is the integral of the function **Simpson's Rule** is used as shown in the figure below:



The area into *n* equal segments of width *Δx*. Note that in Simpson's Rule, *n* must be EVEN. The approximate area is given by the following rule:

$$Area = \int_a^b f(x)dx = \frac{\Delta x}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 \ldots + 4y_{n-1} + y_n)$$

Where $\Delta x = \frac{b-a}{n}$

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<u>Differential equation</u>

Homogeneous form:

$$a\ddot{y} + b\dot{y} + cy = 0$$

Characteristic equation:

$$a\lambda^2 + b\lambda + c = 0$$

Quadratic solutions :

$$\lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

i. If $b^2 - 4ac > 0$, λ_1 and λ_2 are distinct real numbers then the general solution of the differential equation is:

$$y(t) = Ae^{\lambda_1 t} + Be^{\lambda_2 t}$$

A and B are constants.

ii. If $b^2 - 4ac = 0$, $\lambda_1 = \lambda_2 = \lambda$ then the general solution of the differential equation is:

$$y(t) = e^{\lambda t}(A + Bx)$$

A and B are constants.

iii. If $b^2 - 4ac < 0$, λ_1 and λ_2 are complex numbers then the general solution of the differential equation is:

$$y(t) = e^{\alpha t}[Acos(\beta t) + Bsin(\beta t)]$$

$$\alpha = \frac{-b}{2a} \quad and \quad \beta = \frac{\sqrt{4ac - b^2}}{2a}$$

A and B are constants.

<u>Inverse of 2x2 matrices:</u>

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

The inverse of A can be found using the formula:

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

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<u>modelling growth and decay of engineering problem</u>

$C(t) = C_0 e^{kt}$

$k > 0$ gives exponential growth

$k < 0$ gives exponential decay

<u>First order system</u>

$$y(t) = k(1 - e^{-\frac{t}{\tau}})$$

Transfer function: $\frac{k}{\tau s+1}$

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Derivatives table:

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
k, any constant	0
x	1
x^2	$2x$
x^3	$3x^2$
x^n, any constant n	nx^{n-1}
e^x	e^x
e^{kx}	ke^{kx}
$\ln x = \log_e x$	$\frac{1}{x}$
$\sin x$	$\cos x$
$\sin kx$	$k \cos kx$
$\cos x$	$-\sin x$
$\cos kx$	$-k \sin kx$
$\tan x = \frac{\sin x}{\cos x}$	$\sec^2 x$
$\tan kx$	$k \sec^2 kx$
$\text{cosec}\, x = \frac{1}{\sin x}$	$-\text{cosec}\, x \cot x$
$\sec x = \frac{1}{\cos x}$	$\sec x \tan x$
$\cot x = \frac{\cos x}{\sin x}$	$-\text{cosec}^2 x$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$\frac{-1}{\sqrt{1-x^2}}$

Integral table:

$f(x)$	$\int f(x)\,\mathrm{d}x$
k, any constant	$kx + c$
x	$\frac{x^2}{2} + c$
x^2	$\frac{x^3}{3} + c$
x^n	$\frac{x^{n+1}}{n+1} + c$
$x^{-1} = \frac{1}{x}$	$\ln\lvert x\rvert + c$
e^x	$e^x + c$
e^{kx}	$\frac{1}{k}e^{kx} + c$
$\cos x$	$\sin x + c$
$\cos kx$	$\frac{1}{k}\sin kx + c$
$\sin x$	$-\cos x + c$
$\sin kx$	$-\frac{1}{k}\cos kx + c$
$\tan x$	$\ln(\sec x) + c$
$\sec x$	$\ln(\sec x + \tan x) +$
$\operatorname{cosec} x$	$\ln(\operatorname{cosec} x - \cot x) +$
$\cot x$	$\ln(\sin x) + c$
$\cosh x$	$\sinh x + c$
$\sinh x$	$\cosh x + c$
$\tanh x$	$\ln\cosh x + c$
$\coth x$	$\ln\sinh x + c$
$\frac{1}{x^2+a^2}$	$\frac{1}{a}\tan^{-1}\frac{x}{a} + c$

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Laplace table:

$f(t)$	$F(s)$		$f(t)$	$F(s)$
1	$\frac{1}{s}$		$u_c(t)$	$\frac{e^{-cs}}{s}$
t	$\frac{1}{s^2}$		$\delta(t)$	1
t^n	$\frac{n!}{s^{n+1}}$		$\delta(t-c)$	e^{-cs}
e^{at}	$\frac{1}{s-a}$		$f'(t)$	$sF(s)-f(0)$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$		$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos bt$	$\frac{s}{s^2+b^2}$		$(-t)^n f(t)$	$F^{(n)}(s)$
$\sin bt$	$\frac{b}{s^2+b^2}$		$u_c(t)f(t-c)$	$e^{-cs}F(s)$
$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}$		$e^{ct}f(t)$	$F(s-c)$
$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$		$\delta(t-c)f(t)$	$e^{-cs}f(c)$

END OF FORMULA SHEETS

END OF PAPER