[ESS04]

UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

BENG (HONS) IN MECHANICAL ENGINEERING

SEMESTER TWO EXAMINATION 2018/2019

ENGINEERING PRINCIPLES 2

MODULE NO: AME4063 & AME4053

Date: Wednesday 22nd May 2019 **Time: 10:00 – 12:00**

INSTRUCTIONS TO CANDIDATES:

This paper is split into two parts; Part A and Part B. There are THREE questions in Part A and THREE questions in Part B.

Answer FOUR questions in total; TWO questions from Part A and TWO questions from Part B.

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

This examination paper carries a total of 100 marks.

CANDIDATES REQUIRE: **Formula sheet (attached)**

School of Engineering
BEng (Hons) in Mechanical Engineering
Semester Two Examination 2018/2019
Engineering Principles 2
Module No: AME4063 & AME4053

Part A

Q1

a) A flywheel 0.9 m diameter has its initial angular velocity of 6rad/s increased to its final angular velocity with an angular acceleration of 12 rad/s^2 whilst making 100 revolutions.

Calculate:

i) The final angular velocity of the flywheel **(5 marks)**
ii) The time taken for the 100 revolutions **(5 marks)**
iii) The linear acceleration and final linear velocity of a point on the rim of the flywheel

(5 marks)

b) A turbine rotor has a moment of inertia of 1.4 Mgm^2. Determine the acceleration torque required to accelerate the rotor from 26000 rev /min to 2700 rev/min in a time of 2s. .

(10 marks)

Total 25 marks

Q2

a) for the beam cross section shown in Figure Q2a find the centroid.

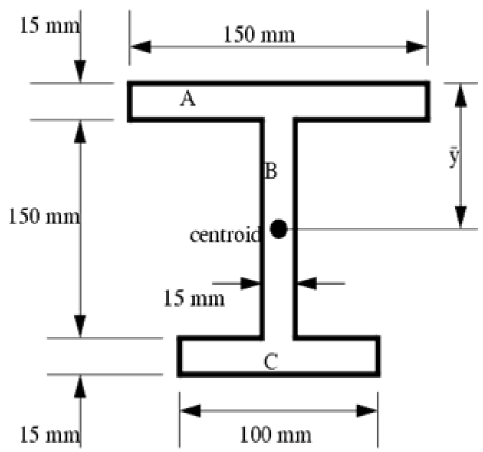


Figure Q2A

(15 marks)

Q2 continues over the page…

PLEASE TURN THE PAGE…..

Q2 continued…

b) Define the moment of inertia and radius of gyration

(10 marks)

Total 25 marks

Q3

a) Find the second moment of area and radius of gyration about the axis XX for the beam section shown in Figure Q3a.

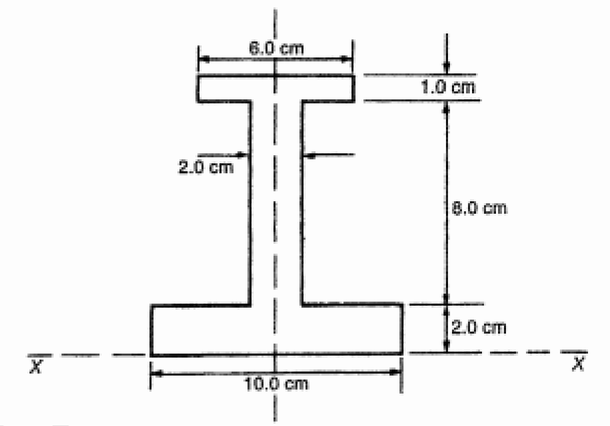


Figure Q3a

(10 marks)

b) A rectangular section beam has a depth of 100mm and width 24 mm and is subject to a bending moment of 2.5kN m. Calculate the maximum stress in the beam .Take E=206 GPa

(8 marks)

C) A solid steel shaft 2m long and 60mm diameter rotates at 200rev/min. Calculate the torque when the maximum shear stress in the shaft is 70 MPa.

(7 marks)

Total 25 marks

END OF PART A

PLEASE TURN THE PAGE FOR PART B…..

PART B

Q4) a) Calculate the derivative of the function f defined by:

$$f(x) = x^2$$

from first principles. **(3 marks)**

b) Calculate the first derivative of the following functions:

i) $5e^{-2x} + 4x^3$ **(2 marks)**

ii) $2\cos(3x + 6)$ **(3 marks)**

iii) $6xe^{-4x}$ **(3 marks)**

iv) $\frac{2x+1}{x^2+2}$ **(3 marks)**

c) Find and classify the stationary points of the curve $y = f(x)$ where the function f is defined by:

$$f(x) = 2x^3 - 21x^2 + 60x + 4$$ **(4 marks)**

d) Consider the following equation:

$$e^{2x} - 8x^2 = 4$$

(i) Show there is a solution to this equation on the interval$[0,2]$. **(2 marks)**

(ii) Use the method of bisection *once* to find a first approximation to the solution of the equation. **(2 marks)**

(iii) Using the approximation calculated in (ii) as your initial value, use the Newton-Raphson method to find a solution to the equation accurate to 2 decimal places. **(3 marks)**

Total: 25 marks

PLEASE TURN THE PAGE…..

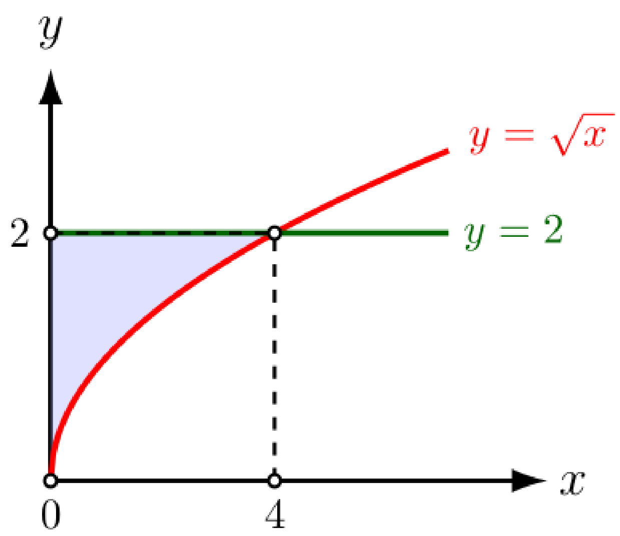
Q5) a) Evaluate the following definite integrals:

(i) $\int_1^2 \left(10x^4 + \frac{1}{2}x^2\right)\, dx$ **(4 marks)**

(ii) $\int_0^\pi x\cos(3x)\, dx$ **(4 marks)**

(iii) $\int_0^2 \cos(2x-4)\, dx$
using the substitution $u = g(x) = 2x - 4$. **(4 marks)**

b) Find the area between the curves $y = 2$, $y = \sqrt{x}$ and the y-axis, as indicated by the blue region in the following diagram: **(5 marks)**



c) Consider the following integral: $\int_0^3 \frac{1}{x^3+10}\, dx$.

Approximate the value of this integral with 6 strips using:

(i) the trapezoidal rule; and **(4 marks)**

(ii) Simpson's rule. **(4 marks)**

Give your answers to 4 decimal places.

Total 25 marks

PLEASE TURN THE PAGE…..

Q6) a) Find the particular solution to the following differential equations:

(i) $$\begin{cases} y' = 2xy \\ y(0) = 4 \end{cases}$$ **(5 marks)**

(ii) $$\begin{cases} xy' + 3y = 4x \\ y(2) = 3 \end{cases}$$ **(5 marks)**

(iii) $$\begin{cases} y' + x = \cos(2x) \\ y(0) = 1 \end{cases}.$$ **(5 marks)**

b) Find the particular solution of the differential equation:

$$\begin{cases} y'' + 7y' + 12y = 0 \\ y(0) = 1 \\ y'(0) = 1 \end{cases}$$ **(10 marks)**

Total: 25 marks

END OF PART B

END OF QUESTIONS

PLEASE TURN THE PAGE FOR FORMULA SHEETS…..

School of Engineering
BEng (Hons) in Mechanical Engineering
Semester Two Examination 2018/2019
Engineering Principles 2
Module No: AME4063 & AME4053

Formula Sheet

2nd Moments of Area

Rectangle $I = \frac{bd^3}{12}$

Circle $I = \frac{\pi d^4}{64}$ Polar $J = \frac{\pi d^4}{32}$

Parallel Axis Theorem

$I_{xx} = I_{GG} + Ah^2$

Bending

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

Torsion

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\vartheta}{\ell}$$

Motion

$v = u + at$ $\omega_2 = \omega_1 + \alpha t$

$v^2 = u^2 + 2as$ $\omega_2^2 = \omega_1^2 + 2\alpha\vartheta$

$s = \left(\frac{u+v}{2}\right)t$ $\vartheta = \left(\frac{\omega_1+\omega_2}{2}\right)t$

$s = ut + \frac{1}{2}at^2$ $\vartheta = \omega_1 t + \frac{1}{2}\alpha t^2$

Speed = $\frac{\text{Distance}}{\text{Time}}$ Acceleration = $\frac{\text{Velocity}}{\text{Time}}$

$s = r\vartheta$
$V = \omega r$
$a = \alpha r$

Torque and Angular

$T = I\alpha$

$I = mk^2$

$P = T\omega$

Energy and Momentum

Potential Energy = mgh

Kinetic Energy

Linear = ½ mv^2

Angular =½ $I\omega^2$

Momentum

Linear = mv

Angular = $I\omega$

Vibrations

Linear Stiffness $k = \frac{F}{\delta}$

Circular frequency $\omega_n = \sqrt{\frac{k}{m}}$

Frequency $f_n = \frac{\omega_n}{2\pi} = \frac{1}{T_n}$

$x = r\cos\omega t$

$v = -\omega\sqrt{r^2 - x^2} = -\omega r\sin\omega t$

$a = -\omega^2 x$

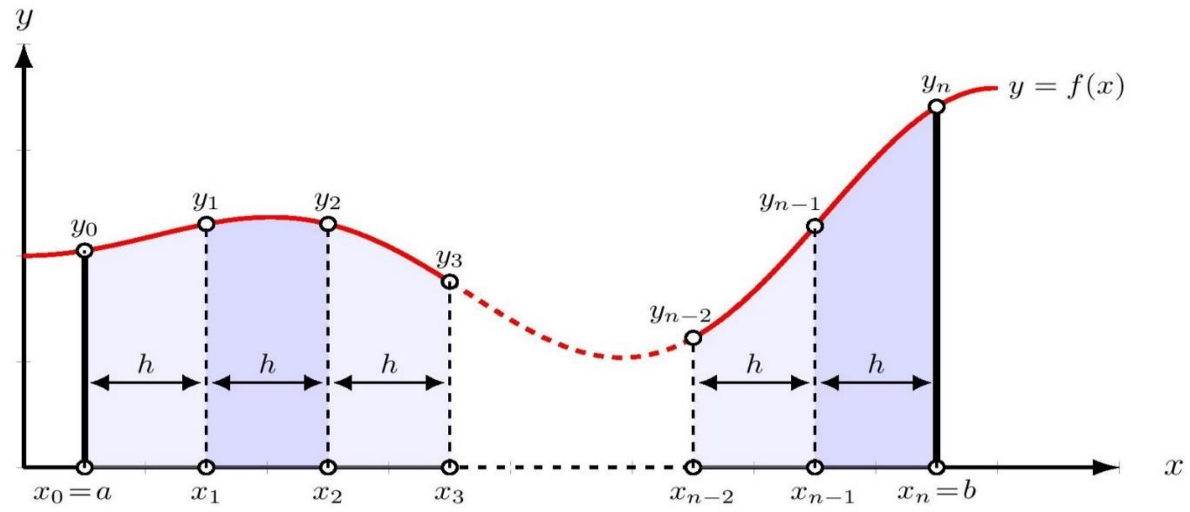
$f = \frac{1}{T}$

$T = \frac{2\pi}{\omega}$

$F = ma$

School of Engineering
BEng (Hons) in Mechanical Engineering
Semester Two Examination 2018/2019
Engineering Principles 2
Module No: AME4063 & AME4053

Numerical Methods



In both approximation rules: $h = \dfrac{b-a}{n}$ where n is the number of strips.

Trapezium Rule for n Strips:

$$\int_a^b f(x)\,dx \approx \frac{1}{2}h\Big[\,y_0 + 2\,(y_1 + y_2 + y_3 + \cdots\cdots + y_{n-1}) + y_n\,\Big]$$

Simpson's Rule for n Strips (where n <u>must</u> be even):

$$\int_a^b f(x)\,dx \approx \frac{1}{3}h\Big[\,y_0 + 4\overbrace{\big(y_1 + y_3 + \cdots + y_{n-1}\big)}^{\text{Odd numbered terms}} + 2\underbrace{\big(y_2 + y_4 + \cdots + y_{n-2}\big)}_{\text{Even numbered terms}} + y_n\Big]$$

Newton-Raphson Method
Approximate solutions to $f(x) = 0$ (i.e. roots of the function f) can be found using the iterative scheme:

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}$$

with $x = x_0$ some (given) initial point.

PLEASE TURN THE PAGE…..

Integration and Differentiation

Differentiation from First Principles

The first derivative of a function $f(x)$ with respect to x is given by:

$$f'(x) = \lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{h} \right).$$

Table of Derivatives and Integrals

In the table below, m, n are any real numbers.

$\int F(x)\,dx$	$F(x)$	$F'(x)$
$\int f(x)\,dx + \int g(x)\,dx$	$f(x) + g(x)$	$f'(x) + g'(x)$
$m \int f(x)\,dx$	$mf(x)$	$mf'(x)$
$mx + C$	m	0
$\frac{x^{n+1}}{n+1} + C \quad (n \neq 1)$	x^n	nx^{n-1}
$\ln(x) + C$	$\frac{1}{x}$	$-\frac{1}{x^2}$
$\frac{1}{m} e^{mx} + C$	e^{mx}	me^{mx}
$x - x\ln(mx) + C$	$\ln(mx)$	$\frac{1}{x}$
$\frac{1}{m} \sin(mx) + C$	$\cos(mx)$	$-m \sin(mx)$
$-\frac{1}{m} \cos(mx) + C$	$\sin(mx)$	$m \cos(mx)$

Rules of Differentiation

PRODUCT RULE: $$\frac{d}{dx}\Big(f(x)g(x) \Big) = f'(x)g(x) + f(x)g'(x)$$

QUOTIENT RULE: $$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

CHAIN RULE: $$\frac{d}{dx}\Big(f(\,g(x)\,) \Big) = g'(x) \cdot f'(\,g(x)\,)$$

Rules of Integration

INTEGRATION BY PARTS: $$\int_{x=a}^{b} f(x)g'(x)\,dx = \Big[f(x)g(x) \Big]_{x=a}^{b} - \int_{x=a}^{b} f'(x)g(x)\,dx$$

INTEGRATION BY SUBSTITUTION: $$\int_{x=a}^{b} F(\,g(x)\,)\,g'(x)\,dx = \int_{u=g(a)}^{g(b)} F(u)\,du$$

with the substitution $u = g(x)$ and where $F'(x) = f(x)$.

Local Maxima and Minima of a Function

A curve defined by $y = f(x)$ in terms of some function f has *stationary points* where $f'(x) = 0$. These are then classified using the *Second Derivative Test*:

Let $x = a$ be a stationary point of $f(x)$ then:

$f''(a) > 0 \implies$ $x = a$ is a local minimum

$f''(a) < 0 \implies$ $x = a$ is a local maximum

$f''(a) = 0 \implies$ the test is inconclusive.

PLEASE TURN THE PAGE……

Differential Equations

First-order ODEs:
The following denote methods of solving first-order ordinary differential equations:

DIRECT INTEGRATION

$$y' = f(x) \implies y = \int f(x)\,dx$$

SEPARATION OF VARIABLES

$$y' = f(x) \cdot g(y) \implies F(y) = \int f(x)\,dx \quad \text{where} \quad F'(y) = \frac{y'}{g(y)}.$$

INTEGRATING FACTOR

$$y' + f(x)y = g(x) \implies y = \frac{1}{M(x)} \int M(x)g(x)\,dx$$

where $M(x) = \exp\left(\int f(x)\,dx\right)$ is the integrating factor.

Second-order ODEs
The solution to the second-order homogeneous differential equation with constant coefficients:

$$y'' + Ay' + B = 0$$

is determined by the roots of its auxiliary equation:

Case	Roots	General Solution
I	Two real: M_1, M_2	$y = Ae^{M_1 x} + Be^{M_2 x}$
II	One real (double) root: M	$y = (A + Bx)e^{Mx}$
III	Complex conjugate pair: $P \pm i\omega$	$y = \Big(A\cos(\omega x) + B\sin(\omega x)\Big)\,e^{Px}$

END OF FORMULA SHEETS

END OF PAPER